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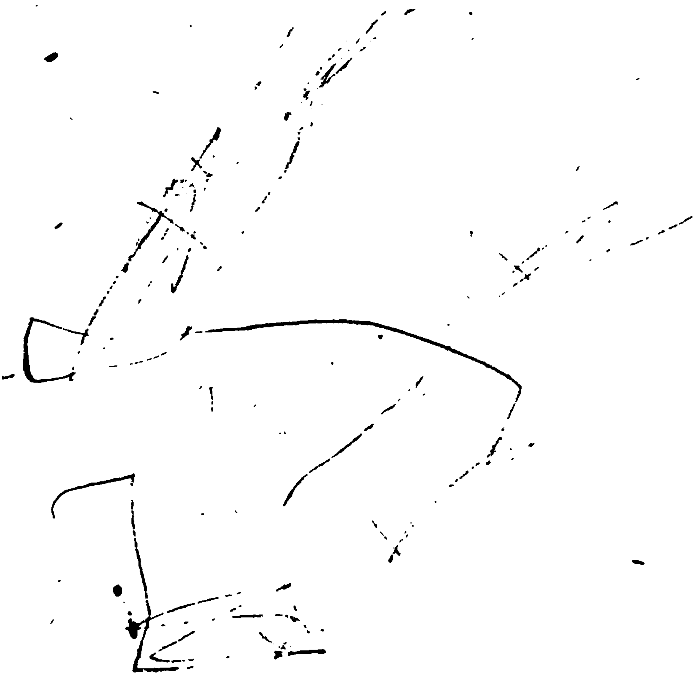

A

KEY

TO THE

NATIONAL ARITHMETIC,

EXHIBITING THE OPERATION OF

THE MORE DIFFICULT QUESTIONS

IN THAT WORK;

FOR THE USE OF TEACHERS ONLY.

BY BENJAMIN GREENLEAF, A. M.
PRINCIPAL OF BRADFORD TEACHERS' SEMINARY.

BOSTON:
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The object of the author, in this publication, is to aid the teacher in communicating his instructions to his pupils, and enable him the more readily to detect any error, which they may have made in the operation of their questions.

Every instructor, who has a large number of scholars under his care, is aware of the fact, that it is a great tax on his time, especially when in school, to examine the operation of many questions of his students; whereas, by the aid of a Key, he may be able, in a few moments, to detect any mistake in the operation, and thereby save much of his time, which may be devoted to more useful purposes. Besides, in the hurry of business, it is often very difficult for the most able arithmetician to recollect, at the moment, all the principles by which some difficult questions are performed; but, by recurring to a Key, his difficulty will be obviated.

The author would recommend the following maxim to every teacher:—Never give a pupil a direct answer to any question he may propose respecting the operation of any problem, nor perform the labor for him, but suggest such principles as will enable him to perform the question himself.

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KEY

TO

GREENLEAF'S ARITHMETIC.

SECTION VI.

COMPOUND ADDITION.

2. (p. 40.) $177.66.3.
3. $2877.87.2.
5. 141£. 10s. 7¾d.
6. 161£. 17s. 7d.
8. 385lb. 7oz. 5dwt. 10gr.
10. 246℔. 10℥. 3ʒ. 2℈. 2gr.
12. 233cwt. 1qr. 6lb. 4oz. 5dr.
14. 317m. 3fur. 18rd. 4yd. 2ft. 6in. 0br.
16. 2513EE. 3qr. 2na. 0in.
18. 317A. 0R. 24p. 13¾ft.
20. 378 cords, 96ft. 1460in.
22. 286hhd. 42gal. 1qt. 1pt.
24. 784hhd. 24gal. 2qt. 0pt.
26. 240ch. 20bu. 2pk. 7qt.
28. 212y. 11m. 29d. 4h.
30. 11S. 2°. $22'$. $42''$.
32. 194m. 6fur. 9ch. 0p. 12li.

COMPOUND SUBTRACTION.

2. (p. 44.) $491.72.9.
4. 435£. 15s. 9¼d.
6. 38lb. 7oz. 14dwt. 13gr.
8. 5℔. 5℥. 0ʒ. 1℈. 17gr.
10. 53cwt. 3qr. 23lb. 14oz.
12. 31EE. 4qr. 2na.
14. 9deg. 4m. 5fur. 37rd. 4yd. 1ft. 4in. 1br.
16. 3A. 1R. 38p. 18yd. 7ft. 34in.
18. 163 cords, 53ft. 1289in.
20. 6hhd. 61gal. 2qt. 1pt.
22. 577hhd. 52gal. 2qt.
24. 26ch. 22bu. 2pk. 4qt.
26. 28y. 11m. 2w. 4d. 21h. 48m. 35sec.
28. 2S. 27°. 21′. 54″.
30. 12m. 6fur. 8ch. 0p. 24li.

SECTION VII.

REDUCTION DESCENDING.

2. (p. 52.) $127 \times 20 + 15 = 2555 \times 12 + 8 = 30668 \times 4 = 122672$ Ans.

3. $28 \times 20 + 19 = 579 \times 12 + 11 = 6959 \times 4 + 3 = 27839$ Ans.

4. $378 \times 20 = 7560 \times 12 = 90720$ Ans.

5. $28 \times 12 + 11 = 347 \times 20 + 12 = 6952 \times 24 + 15 = 166863$ Ans.

6. $17 \times 12 = 204 \times 20 + 12 = 4092$ Ans.

7. $3 \times 12 + 11 = 47 \times 20 = 940 \times 24 = 22560$ Ans

8. $23 \times 12 = 276 \times 8 = 2208 \times 3 = 6624$ Ans.

9. $3 \times 20 + 16 = 76 \times 4 + 2 = 306 \times 28 + 18 = 8586 \times 16 = 137376$ Ans.

10. $2 \times 20 + 17 = 57 \times 4 + 3 = 231 \times 28 + 16 = 6484 \times 16 + 15 = 103759 \times 16 + 13 = 1660157$ Ans.

11. $57 \times 4 = 228 \times 4 = 912$ Ans.

12. $83947 \times 5 + 4 = 419739 \times 4 = 1678956$ Ans.

13. $2263 \times 3 + 2 = 6791$ Ans.

14. $79 \times 8 = 632 \times 40 = 25280 \times 16\frac{1}{2} = 417120$ Ans.

15. $396 \times 40 = 15840 \times 16\frac{1}{2} = 261360 \times 12 = 3136320$ Ans.

16. $30 \times 8 = 240 \times 40 = 9600 \times 16\frac{1}{2} = 158400 \times 12 = 1900800$ Ans.

17. $360 \times 69\frac{1}{2} = 25020 \times 8 = 200160 \times 40 = 8006400 \times 16\frac{1}{2} = 132105600 \times 12 = 1585267200 \times 3 = 4755801600$ Ans.

18. $403 \times 8 + 7 = 3231 \times 40 + 35 = 129275 \times 5\frac{1}{2} + 2 = 711014\frac{1}{2} \times 3 = 2133043\frac{1}{2} \times 12 = 25596522 \times 3 + 1 = 76789567$ Ans.

19. $413 \times 3 + 2 = 1241 \times 8 + 2 = 9930 \times 40 + 38 = 397238 \times 5\frac{1}{2} + 1 = 2184810 \times 3 = 6554430 \times 12 + 1 = 78653167$ Ans.

20. $144 \times 8 + 1 = 1153 \times 40 + 8 = 46128 \times 5\frac{1}{2} + 1 = 253705 \times 3 + 1 = 761116$ Ans.

21. $1051 \times 3 + 2 = 3155 \times 12 + 5 = 37865$ Ans.

22. $3576 \times 40 + 12 = 143052 \times 5\frac{1}{2} + 3 = 786789$ Ans.

23. $25 \times 160 = 4000 \times 272\frac{1}{4} = 1089000$ Ans.

24. $365 \times 640 = 233600 \times 160 = 37376000$ Ans.

25. $196563942 \times 640 = 125800922880 \times 160 = 20128147660800 \times 272\frac{1}{4} = 5479888200652800 \times 144 = 789{,}103{,}900{,}894{,}003{,}200$ Ans.

26. $10 \times 4 + 3 = 43 \times 40 + 38 = 1758 \times 30\frac{1}{4} + 6 = 53185\frac{1}{2} \times 9 + 5 = 478674\frac{1}{2} + 72$in. or $\frac{1}{2}$ft. $= 478675$ Ans.

27. $2 \times 40 = 80 \times 30\frac{1}{4} + 24 = 2444 \times 9 + 3 = 21999 \times 144 = 3167856$ Ans.

28. $1 \times 4 + 3 = 7 \times 40 + 34 = 314 \times 30\frac{1}{4} + 27 = 9525\frac{1}{2} \times 9 + 4 = 85733\frac{1}{2} \times 144 + 54 = 12345678$ Ans.

29. $17 \times 128 = 2176 \times 1728 = 3760128$ Ans.

30. $19 \times 40 = 760 \times 1728 = 1313280$ Ans.

31. $128 \times 128 = 16384$ Ans.

32. $4899 \times 63 + 4 = 308641 \times 4 + 3 = 1234567$ Ans.

33. $1224 \times 2 + 1 = 2449 \times 2 + 1 = 4899 \times 63 + 19 = 308656 \times 4 + 1 = 1234625 \times 2 = 2469250 \times 4 + 1 = 9877001$ Ans.

34. $790 \times 2 = 1580 \times 63 + 58 = 99598 \times 4 = 398392 \times 2 + 1 = 796785$ Ans.

35. $460 \times 2 + 1 = 921 \times 54 + 31 = 49765$ Ans.

36. $36 \times 54 + 26 = 1970 \times 4 + 3 = 7883 \times 2 + 1 = 15767$ Ans.

37. $16 \times 40 = 640 \times 1728 = 1105920$ Ans.

38. $365\frac{1}{4} \times 24 = 8766 \times 60 = 525960 \times 60 = 31557600$; $2348 + 1835 = 4183 \times 31557600 = 132005440800$ Ans.

39. June 13 + July 31 + August 31 + September 30 + October 31 + November 30 + December 31 + January 31 + February 16 + 365 + 365 = 974 days, Ans.

40. $676 \times 36 = 24336 \times 4 = 97344$ pecks, Ans.

41. $657 \times 10 = 6570$ mills, Ans.

42. $3165 \times 100 = 316500$ mills, Ans.

43. $63 \times 100 = 6300$ cents, Ans.

44. $27 \times 10 \times 100 \times 10 = 270000$ mills, Ans.

REDUCTION ASCENDING.

2. (p. 54.) $122672 \div 4 = 30668$d. $\div 12 = 2555$s. 8d. $\div 20 = 127$£. 15s. 8d. Ans.

3. $27839 \div 4 = 6959\frac{3}{4}$d. $\div 12 = 579$s. 11d. $\div 20 = 28$£. 19s. $11\frac{3}{4}$d. Ans.

4. $90720 \div 12 = 7560$s. $\div 20 = 378$£. Ans.

5. 166863 ÷ 24 = 6952dwt. 15gr. ÷ 20 = 347oz. 12dwt. ÷ 12 = 28lb. 11oz. 12dwt. 15gr. Ans.

6. 4092dwt. ÷ 20 = 204oz. 12dwt. ÷ 12 = 17lb. 0oz. 12dwt. Ans.

7. 22560 ÷ 24 = 940dwt. ÷ 20 = 47oz. ÷ 12 = 3lb. 11oz. Ans.

8. 6624 ÷ 3 = 2208℈. ÷ 8 = 276ʒ. ÷ 12 = 23℔. Ans.

9. 137376 ÷ 16 = 8586lb. ÷ 28 = 306qr. 18lb. ÷ 4 = 76cwt. 2qr. ÷ 20 = 3 tons, 16cwt. 2qr. 18lb. Ans.

10. 1660157 ÷ 16 = 103759oz. 13dr. ÷ 16 = 6484lb. 15oz. ÷ 28 = 231qr. 16lb. ÷ 4 = 57cwt. 3qr. ÷ 20 = 2T. 17cwt. 3qr. 16lb. 15oz. 13dr. Ans.

11. 912 ÷ 4 = 228qr. ÷ 4 = 57yd. Ans.

12. 1678956 ÷ 4 = 419739gr. ÷ 5 = 83947EE. 4qr. Ans.

13. 6791 ÷ 3 = 2263EF. 2qr. Ans.

14. 417120 ÷ 16½ = 25280rd. ÷ 40 = 632fur. ÷ 8 = 79m. Ans.

15. 3136320 ÷ 12 = 261360ft. ÷ 16½ = 15840rd. ÷ 40 = 396fur. Ans.

16. 1900800 ÷ 12 = 158400ft. ÷ 16½ = 9600rd. ÷ 40 = 240fur. ÷ 8 = 30m. Ans.

17. 4755801600 ÷ 3 = 1585267200in. ÷ 12 = 132105600ft. ÷ 16½ = 8006400rd. ÷ 40 = 200160fur. ÷ 8 = 25020m. ÷ 69½ = 360deg. Ans.

18. 76789567 ÷ 3 = 25596522in. 1br. ÷ 12 = 2133043ft. 6in. ÷ 3 = 711014yd. 1ft. ÷ 5½ = 129275rd. 1½yd. ÷ 40 = 3231fur. 35rd. ÷ 8 =

403m. 7fur. 35rd. 1½yd. 1ft. 6in. 1br.
½yd. = 1ft. 6in. 0br.
403m. 7fur. 35rd. 2yd. 0ft. 0in. 1br. Ans.

19. 78653167 ÷ 12 = 6554430ft. 7in. ÷ 3 = 2184810yd. ÷ 5½ = 397238rd. 1yd. ÷ 40 = 9930fur. 38rd. ÷ 8 = 1241m. 2fur. ÷ 3 = 413L. 2m. 2fur. 38rd. 1yd. 0ft. 7in. Ans.

20. 761116÷3=253705yd. 1ft. ÷5½=46128rd. 1yd. ÷40=1153fur. 8rd.÷8=144m. 1fur. 8rd. 1yd. 1ft. Ans.

21. 37865÷12=3155ft. 5in.÷3=1051yd. 2ft. Ans.

22. 786789 ÷ 5½ = 143052rd. 3yd. ÷ 40 = 3576fur. 12rd. 3yd. Ans.

23. 1089000÷272¼=4000rd.÷160=25A. Ans.

24. 37376000÷160=233600A.÷640=365 square miles, Ans.

25. 789,103,900,894,003,200÷144=5479888200652800ft.÷272¼=20128147660800rd.÷160=125800922880A. ÷640=196563942 square miles, Ans.

26. 478675÷9=53186yd. 1ft.÷30¼=1758rd. 6½yd. ÷40=43R. 38rd.÷4=

10A. 3R. 38rd. 6½yd. 1ft.
½yd.=4ft. 72in.
10A. 3R. 38rd. 6yd. 5ft. 72in. Ans.

27. 3167856 ÷ 144 = 21999ft. ÷ 9=2444yd. 3ft. ÷ 30¼=80rd. 24yd. ÷40=2R. 0rd. 24yd. 3ft. Ans.

28. 12345678÷144=85733ft. 126in.÷9 = 9525yd. 8ft.÷30¼=314p. 26½yd. ÷40=7R. 34p.÷4=

1A. 3R. 34p. 26½yd. 8ft. 126in.
½yd.=4ft. 72in.
1A. 3R. 34p. 27yd. 4ft. 54in. Ans.

29. 3760128÷1728=2176ft.÷128=17 cords, Ans.

30. 1313280÷1728=760ft.÷40=19 tons, Ans.

31. 16384÷128=128 cords, Ans.

32. 1234567÷4 = 308641gal. 3qt.÷63 = 4899hhd. 4gal. 3qt. Ans.

33. 9877001÷4=2469250pt. 1gi.÷2=1234625qt. ÷ 4 = 308656gal. 1qt. ÷ 63 = 4899hhd. 19gal. ÷ 2 = 2449pi. 1hhd.÷2=1224T. 1pi. 1hhd. 19gal. 1qt. 0pt. 1gi. Ans.

34. 796785÷2=398392qt. 1pt.÷4=99598qt.÷63

= 1580hhd. 58gal. ÷ 2 = 790pi. 0hhd. 58gal. 0qt. 1pt. Ans.

35. 49765 ÷ 54 = 921hhd. 31gal. ÷ 2 = 460 butts, 1hhd. 31gal. Ans.

36. 15767 ÷ 2 = 7883qt. 1pt. ÷ 4 = 1970gal. 3qt. ÷ 54 = 36hhd. 26gal. 3qt. 1pt. Ans.

37. 1105920 ÷ 1728 = 640ft. ÷ 40 = 16 tons, Ans.

38. 132005440800 ÷ 60 = 2200090680m. ÷ 60 = 36668178h. ÷ 24 = $1527840\frac{3}{4}$ days, ÷ $365\frac{1}{4}$ = 4183 years.

39. 974 ÷ 365 = 2 years, 244 days, Ans.

40. 97344 ÷ 4 = 24336bu. ÷ 36 = 676chald. Ans.

41. 6570 ÷ 10 = 657 cents, Ans.

42. 316500 ÷ 100 = 3165 dimes, Ans.

43. 6300 ÷ 100 = $63 Ans.

44. 270000 ÷ 10 = 27000 cts. ÷ 100 = $270 ÷ 10 = 27 Eag. Ans.

COMPOUND REDUCTION.

1. (p. 55.) 57£. 15s. = 1155s. ÷ 6 = $192.50.

2. 67£. 14s. 9d. = 16257d.; 6s. 7d. = 79d.; 16257 ÷ 79 = 205cr. 62d. = 5s. 2d. Ans.

3. $678 × 6 = 4068s. ÷ 20 = 203£. 8s. Ans.

4. 761 × 4 = 3044qr. ÷ 5 = 608EE. 4qr. Ans.

5. 61 × 3 = 183qr. ÷ 4 = 45yd. 3qr. Ans.

6. 63 × 4 = 252 × 2 = 504 ÷ 3 = 168 bottles, Ans.

7. 15 × 1760 = 26400yd. × 3 = 79200ft. × 12 = 950400in.; 2ft. 8in. = 32in. 950400 ÷ 32 = 29700 steps, Ans.

8. 2oz. 12dwt. = 52dwt.; 5lb. 2oz. 8dwt. = 1248dwt. ÷ 52 = 24 spoons, Ans.

9. 14ft. 9in. = 177in.; 436 × 1760 = 767360yd. × 3 = 2302080ft. × 12 = 27624960in.; 27624960 ÷ 177 = $156073\frac{39}{177}$ times, Ans.

10. 123 × 78 = 9594 hills, × 4 = 38376 ears, ÷ 8 = 4797qt. ÷ 8 = 599pk. 5qt. ÷ 4 = 149bu. 3pk. 5qt. Ans.

11. 5yd. 2qr. 3na. = 91na.; 182yd. = 2912na. ÷ 91 = 32 suits, Ans.

12. 5dwt. 10gr. = 130gr.; 3lb. 1oz. 2dwt. 2gr. = 17810gr. ÷ 130 = 137 rings, Ans.

13. 18 ÷ 3 = 6 × 4 = 24in.; 56 × 60 × 144 = 483840in. ÷ 24 = 20160 shingles, Ans.

14. 56 × 25 × 2 = 2800 feet, × 6 = 16800 shingles, Ans.

15. 22m. 3fur. 17rd. = 7177rd.; 25000m. = 8000000rd. ÷ 7177 = 1114$\frac{4822}{7177}$ days, Ans.

16. 7lb. 10oz. = 122oz.; 10cwt. 3qr. 16lb. = 19520oz. ÷ 122 = 160 weeks, Ans.

17. 3T. 17cwt. 3qr. 18lb. = 8726lb. × 7 = 61082d. = 254£. 10s. 2d. Ans.

18. 5cwt. 1qr. 10lb. = 598lb. × 4$\frac{1}{2}$ = 2691d. = 11£. 4s. 3d. Ans.

19. 63 × 4 × 7 × 9 = $158.76 Ans.

20. 54 × 4 × 2 × 15 × 3 = $194.40 Ans.

21. 73 × 4 × 8 = 2336qt. × 2 = $46.72 Ans.

22. 29 × 57 = 1653yd. × 15 = $247.95 Ans.

23. 6cwt. 2qr. 11lb. + 5cwt. 3qr. 16lb. + 7cwt. 0qr. 7lb. + 3cwt. 1qr. 17lb. = 22cwt. 3qr. 23lb. = 2571lb. × 15 = $385.65 Ans.

24. 12cwt. = 1344lb.; 3cwt. 2qr. 11lb. + 4cwt. 1qr. 15lb. = 7cwt. 3qr. 26lb. = 894lb.; 1344 − 894 = 450lb. × 15 = $67.50 Ans.

25. 2cwt. 1qr. 7lb. + 3cwt. 2qr. 15lb. + 2cwt. 0qr. 20lb. + 5cwt. 3qr. 17lb. = 14cwt. 0qr. 3lb. = 1571lb. × 37$\frac{1}{2}$ = 589.12\frac{1}{2}$ Ans.

26. 87 × 63 = 5481gal. × 33 = $1808.73 Ans.

27. 10gal. 1qt. 1pt. 3gi. = 335gi.; 63 × 4 = 252qt. × 2 = 504pt. × 4 = 2016gi. — 335gi. = 1681gi. × 6 = $100.86 Ans.

28. 100A. 3R. 15p. + 161A. 2R. 28p. + 360A. 3R. 5p. = 623A. 1R. 8p.; 112A. 3R. 30p. + 316A. 1R. 18p. + 168A. 3R. 13p. = 598A. 0R. 21p.; 623A. 1R. 8p. — 598A. 0R. 21p. = 25A. 0R. 27p. = 4027p. × 1.35 = $5436.45 Ans.

29. 87gal. 1qt. — 13gal. = 74gal. 1qt. = 2376gi. × .01 = $23.76 Ans.

30. 25cwt. 0qr. 17lb. + 37cwt. 2qr. 17lb. + 18cwt. 3qr. 14lb. + 37cwt. 1qr. 17lb. = 119cwt. 0qr. 9lb. = 13337lb. × .02 = $266.74 Ans.

SECTION XI.

VULGAR FRACTIONS.

CASE VIII.

2. (p. 87.) $\frac{4}{5} \times \frac{1}{4} \times \frac{1}{12} \times \frac{1}{20} = \frac{4}{4800} = \frac{1}{1200}$ Ans
3. $\frac{3}{5} \times \frac{1}{24} \times \frac{1}{20} \times \frac{1}{12} = \frac{3}{28800} = \frac{1}{9600}$ Ans.
4. $\frac{5}{9} \times \frac{1}{3} \times \frac{1}{8} \times \frac{1}{12} = \frac{5}{2592}$ Ans.
5. $\frac{6}{11} \times \frac{1}{16} \times \frac{1}{28} \times \frac{1}{4} = \frac{6}{19712} = \frac{3}{9856}$ Ans.
6. $\frac{3}{4} \times \frac{1}{28} \times \frac{1}{4} \times \frac{1}{20} = \frac{3}{8960}$ Ans.
7. $\frac{1}{5} \times \frac{1}{2\frac{1}{4}} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{225}$ Ans.
8. $\frac{4}{7} \times \frac{1}{12} \times \frac{1}{16\frac{1}{2}} \times \frac{1}{40} \times \frac{1}{8} = \frac{4}{443520} = \frac{1}{110880}$ Ans.
9. $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{12} \times \frac{1}{16\frac{1}{2}} \times \frac{1}{40} \times \frac{1}{8} \times \frac{1}{3} = \frac{1}{1140480}$ Ans.
10. $\frac{3}{4} \times \frac{1}{144} \times \frac{1}{272\frac{1}{4}} \times \frac{1}{40} \times \frac{1}{4} = \frac{3}{25090560}$ Ans.
11. $\frac{7}{8} \times \frac{1}{4} \times \frac{1}{63} \times \frac{1}{4} = \frac{1}{1152}$ Ans.
12. $\frac{3}{5} \times \frac{1}{2} \times \frac{1}{8} \times \frac{1}{4} = \frac{3}{320}$ Ans.
13. $\frac{1}{8} \times \frac{1}{60} \times \frac{1}{24} \times \frac{1}{365\frac{1}{4}} = \frac{1}{4207680}$ Ans.

CASE IX.

2. (p. 88.) $\frac{1}{1200} \times \frac{20}{1} \times \frac{12}{1} \times \frac{4}{1} = \frac{960}{1200} = \frac{4}{5}$ Ans.
3. $\frac{1}{960} \times \frac{12}{1} \times \frac{20}{1} \times \frac{24}{1} = \frac{576}{960} = \frac{3}{5}$ Ans.
4. $\frac{5}{2592} \times \frac{12}{1} \times \frac{8}{1} \times \frac{3}{1} = \frac{1440}{2592} = \frac{5}{9}$ Ans.
5. $\frac{3}{9856} \times \frac{4}{1} \times \frac{28}{1} \times \frac{16}{1} = \frac{5376}{9856} = \frac{6}{11}$ Ans.
6. $\frac{3}{8960} \times \frac{20}{1} \times \frac{4}{1} \times \frac{28}{1} = \frac{6720}{8960} = \frac{3}{4}$ Ans.
7. $\frac{1}{225} \times \frac{5}{1} \times \frac{4}{1} \times \frac{2\frac{1}{4}}{1} = \frac{45}{225} = \frac{1}{5}$ Ans.
8. $\frac{1}{110880} \times \frac{8}{1} \times \frac{40}{1} \times \frac{16\frac{1}{2}}{1} \times \frac{12}{1} = \frac{63360}{110880} = \frac{4}{7}$ Ans.
9. $\frac{1}{1140480} \times \frac{3}{1} \times \frac{8}{1} \times \frac{40}{1} \times \frac{16\frac{1}{2}}{1} \times \frac{12}{1} \times \frac{3}{1} = \frac{570240}{1140480}$ $= \frac{1}{2}$ Ans.
10. $\frac{3}{250905560} \times \frac{4}{1} \times \frac{40}{1} \times \frac{272\frac{1}{4}}{1} \times \frac{144}{1} = \frac{188179200}{250905560} = \frac{3}{4}$ Ans.
11. $\frac{1}{1152} \times \frac{4}{1} \times \frac{63}{1} \times \frac{4}{1} = \frac{1008}{1152} = \frac{7}{8}$ Ans.
12. $\frac{3}{320} \times \frac{4}{1} \times \frac{8}{1} \times \frac{2}{1} = \frac{192}{320} = \frac{3}{5}$ Ans.
13. $\frac{1}{4207680} \times \frac{365\frac{1}{4}}{1} \times \frac{24}{1} \times \frac{60}{1} = \frac{525960}{4207680} = \frac{1}{8}$ Ans.

CASE X.

2.

	s.	d.
	1	0
		7
24)	7	0
Ans.		$3\frac{1}{2}$

3.

	s.	d.	qr.
	28	0	0
			7
9)	196	0	0
Ans.	21	9	$1\frac{1}{3}$

4.

	cwt.	qr.	lb.	oz.	dr.
	1	0	0	0	0
					7
11)	7	0	0	0	0
Ans.		2	15	4	$5\frac{9}{11}$

5.

	lb.	oz.	dr.
	1	0	0
			4
9)	4	0	0
Ans.		7	$1\frac{7}{9}$

6.

	lb.	oz.	dwt.	gr.
	1	0	0	0
				8
9)	8	0	0	0
Ans.		10	13	8

7.

	℔.	℥.	ʒ.	℈.	gr.
	1	0	0	0	0
					4
13)	4	0	0	0	0
Ans.		3	5	1	$12\frac{4}{13}$

8.

	yd.	qr.	na.	in.
	1	0	0	0
				7
13)	7	0	0	0
Ans.	2	0	$1\frac{5}{13}$	

9.

	EE.	qr.	na.	in.
	1	0	0	0
				5
9)	5	0	0	0
Ans.	2	3	$0\frac{1}{4}$	

10.

	m.	fur.	rd.	ft.	in.
	1	0	0	0	0
					11
13)	11	0	0	0	0
Ans.		6	30	12	$8\frac{4}{13}$

11.

	fur.	rd.	ft.	in.
	1	0	0	0
				8
9)	8	0	0	0
Ans.		35	9	2

12.

	A.	R.	p.	yd.	ft.	in.
	1	0	0	0	0	0
						7
13)	7	0	0	0	0	0
Ans.		2	6	4	5	$127\frac{5}{13}$

13.

	p.	ft.	in.
	1	0	0
			9
17)	9	0	0
Ans.		144	$19\frac{1}{17}$

14.

	cord.	ft.	in.
	1	0	0
			1
13)	1	0	0
Ans.		9	$1462\frac{2}{13}$

15.

	hhd.	gal.	qt.	pt.	gi.
	1	0	0	0	0
					2
19)	2	0	0	0	0
Ans.		6	2	1	$0\frac{4}{19}$

16.

	hhd.	gal.
	1	0
		7
9)	7	0
Ans.		42

17.

	year.	da.	h.	m.	sec.
	1	0	0	0	0
					11
23)	11	0	0	0	0
Ans.		174	16	26	$5\frac{5}{23}$

18.

$$7\frac{3\frac{3}{11}}{4\frac{2}{5}}=7\tfrac{90}{121}=\$7.74\tfrac{46}{121}\text{ Ans.}$$

NOTE.—We multiply each of the fractions, $\frac{3}{11}$ and $\frac{2}{5}$, by 55, because it is the multiple of 11 and 5, the denominators.

CASE XI.

7. (p. 90.) 1s. = 48qr.; $3\frac{1}{2}$d. = 14qr.; $\frac{14}{48}=\frac{7}{24}$ Ans.

8. $$\frac{21\times12+9=261\times4+1=1045\times3+1=3136}{28\times12\times4\times3=4032}=\tfrac{7}{9}\text{ Ans.}$$

9. $$\frac{2\times28+15=71\times16+4=1140\times16+5=18245\times11+9=200704}{4\times28\times16\times16\times11=315392}$$
$$=\tfrac{7}{11}\text{ Ans.}$$

10. $$\frac{7\times16+1=113\times9+7=1024}{16\times16\times9=2304}=\tfrac{4}{9}\text{ Ans.}$$

11. $$\frac{10\times20+13=213\times24+8=5120}{12\times20\times24=5760}=\tfrac{8}{9}\text{ Ans.}$$

12. $$\frac{3\times8+5=29\times3+1=88\times20+12=1772\times13+4=23040}{12\times8\times3\times20\times13=74880}=\tfrac{4}{13}\text{ Ans.}$$

13. $$\frac{2\times4=8\times2\frac{1}{4}+1=19\times13+5=252}{4\times4\times2\frac{1}{4}\times13=468}=\tfrac{7}{13}\text{ Ans.}$$

14. $$\frac{2\times4+3=11\times2\frac{1}{4}=24\frac{3}{4}\times4+1=100}{5\times4\times2\frac{1}{4}\times4=180}=\tfrac{5}{9}\text{ Ans.}$$

15. $$\frac{\left.\begin{array}{r}6\times40+30=270\times16\frac{1}{2}+12=4467\times12+8\\=53612\times3=160836\times13+12\end{array}\right\}=2090880}{8\times40\times16\frac{1}{2}\times12\times3\times13=2471040}=\tfrac{11}{13}\text{ Ans.}$$

16. $$\frac{35\times16\frac{1}{2}+9=586\frac{1}{2}\times12+2=7040}{40\times16\frac{1}{2}\times12=7920}=\tfrac{8}{9}\text{ Ans.}$$

17. $$\frac{\left.\begin{array}{r}2\times40+6=86\times30\frac{1}{4}+4=2605\frac{1}{2}\times9+5=\\23454\frac{1}{2}\times144+127=3377575\times13+5\end{array}\right\}=43908480}{4\times40\times30\frac{1}{4}\times9\times144\times13=81544320}=\tfrac{7}{13}\text{ Ans.}$$

18. $$\frac{144\times144+19=20755\times17+1=352836}{272\frac{1}{4}\times144\times17=666468}=\tfrac{9}{17}\text{ Ans.}$$

19. $$\frac{9\times1728+1462=17014\times13+2=221184}{128\times1728\times13=2875392}=\tfrac{1}{13}\text{ Ans.}$$

20. $$\frac{6\times4+2=26\times2+1=53\times4=212\times19+4=4032}{63\times4\times2\times4\times19=38304}=\tfrac{2}{19}\text{ Ans.}$$

21. $\frac{42}{63} = \frac{7}{9}$ Ans.

22. $$\frac{174 \times 24 + 16 = 4192 \times 60 + 26 = 251546 \times 60 + 5 = 15092765 \times 23 + 5 = 347133600}{365\frac{1}{4} \times 24 \times 60 \times 60 \times 23 = 725824800}$$

$= \frac{11}{23}$ Ans.

SECTION XII.

ADDITION OF VULGAR FRACTIONS.

CASE IV.

(p. 95.)

2.

	£.	s.	d.
$\frac{4}{9}$ of a £. =	0	8	$10\frac{2}{3}$
$\frac{3}{7}$ of a £. =	0	8	$6\frac{6}{7}$
$\frac{2}{5}$ of a s. =	0	0	$4\frac{4}{5}$
Ans.	0	17	$10\frac{34}{105}$

3.

	cwt.	qr.	lb.
$\frac{7}{11}$ of a ton =	12	2	$25\frac{5}{11}$
$\frac{17}{22}$ of a cwt. =		3	$2\frac{6}{11}$
Ans.	13	2	0

4.

	qr.	na.	in.
$\frac{3}{5}$ of yd. =	2	1	$1\frac{7}{20}$
$\frac{1}{7}$ of an EE. =		2	$1\frac{13}{14}$
$\frac{6}{7}$ of a qr. =		3	$0\frac{27}{28}$
Ans.	3	3	$1\frac{138}{140}$

5.

	fur.	rd.	ft.	in.	br.
$\frac{7}{11}$ of a mile =	5	3	10	6	0
$\frac{4}{13}$ of a furlong =		12	5	0	$2\frac{10}{13}$
$\frac{9}{22}$ of a yard =			1	2	$2\frac{2}{11}$
	5	15	16	9	$1\frac{136}{143}$ =
Ans.	5	16	0	3	$1\frac{136}{143}$

6.

	A.	R.	p.	ft.	in.
$\frac{5}{7}$ of an A. =	0	2	34	77	$113\frac{1}{7}$
$\frac{2}{3}$ of an A. =	0	2	26	181	72
$\frac{13}{14}$ of an A. =	0	3	28	155	$82\frac{2}{7}$
	2	1	9	$141\frac{3}{4}$	$123\frac{3}{7}$ =
Ans.	2	1	9	142	$87\frac{3}{7}$

7.

	m.	fur.	rd.	ft.	in.
$18\frac{3}{7}$ miles =	18	3	17	2	$4\frac{2}{7}$
$23\frac{11}{14}$ miles =	23	6	11	7	$0\frac{6}{7}$
$19\frac{1}{21}$ miles =	19	0	15	3	$11\frac{1}{7}$
Ans.	61	2	3	13	$4\frac{2}{7}$

8.

	gal.	qt.	pt.	gi.
$\frac{11}{12}$ of a gal. =	0	3	1	$1\frac{1}{3}$
$\frac{1}{12}$ of a hhd. =	5	1	0	0
Ans.	6	0	1	$1\frac{1}{3}$

9.

	d.	h.	m.
$\frac{5}{16}$ of a week =	2	4	30
$\frac{1}{5}$ of a day =		4	48
Ans.	2	9	18

10.

	ft.	in.
$\frac{3}{4}$ of a square ft. =	0	108
$\frac{1}{2}$ a foot square =		36
Ans.	1	0

11.

	rd.	ft.	in.
	11	16	5
			6
	11	16	11 =
Ans.	12	0	5

SECTION XIII.

SUBTRACTION OF VULGAR FRACTIONS.

CASE V.

(p. 100.)

2.

	cwt.	qr.	lb.	oz.	dr.
$\frac{2}{31}$ of a ton =	1	1	4	8	$4\frac{4}{31}$
$\frac{4}{5}$ of a cwt. =		3	5	9	$9\frac{3}{5}$
Ans.	0	1	26	14	$10\frac{82}{155}$

3.

	qr.	na.	in.
$\frac{7}{8}$EE. =	4	1	$1\frac{1}{8}$
$\frac{2}{7}$ of a yd. =	1	0	$1\frac{2}{7}$
Ans.	3	0	$2\frac{5}{56}$

4.

	fur.	rd.	ft.	in.
$\frac{2}{9}$ of a mile =	1	31	1	10
$\frac{7}{11}$ of a fur. =		25	7	6
	1	5	$10\frac{1}{2}$	4=
Ans.	1	5	10	10

5.

	m.	fur.	rd.	ft.	in.	br.
$\frac{5}{7}$ deg. =	49	5	5	11	9	$1\frac{2}{7}$
$\frac{3}{5}$ mile =		4	32	0	0	0
Ans.	49	0	13	11	9	$1\frac{2}{7}$

6.

	R.	p.	yd.	ft.	in.
$\frac{4}{11}$A. =	1	18	5	4	72
$\frac{4}{9}$ rod =	0	0	13	4	0
	1	17	$22\frac{1}{4}$	0	72
			$\frac{1}{4}$=	2	36
Ans	1	17	22	2	108

7.

	ft.	in.
$\frac{9}{10}$ cord =	115	$345\frac{3}{5}$
$\frac{2}{11}$ cord =	23	$471\frac{3}{11}$
Ans.	91	$1602\frac{18}{55}$

8.

	gal.	qt.	pt.	gi.
$\frac{7}{13}$hhd. =	33	3	1	$1\frac{7}{13}$
$\frac{4}{5}$ of $\frac{7}{13}$hhd. = $\frac{28}{65}$ =	27	0	1	$0\frac{28}{65}$
Ans.	6	3	0	$1\frac{7}{65}$

9.

	m.	fur.	rd.	ft.	in.	br.
72m. =	72	0	0	0	0	0
$\frac{4}{7}$ of 72m. =	41	1	5	11	9	$1\frac{2}{7}$
	30	6	34	$4\frac{1}{2}$	2	$1\frac{5}{7}$
				$\frac{1}{2}$ =	6	0
Ans.	30	6	34	4	8	$1\frac{5}{7}$

10.

	day.	h.	m.	sec.
$\frac{2}{7}$ year =	104	8	34	$17\frac{1}{7}$
$\frac{4}{9}$ week =	3	2	40	0
Ans.	101	5	54	$17\frac{1}{7}$

11.

	R.	p.	yd.	ft.	in.
$\frac{4}{11}$A. =	1	18	5	4	72
$\frac{1}{2}$ft. =					72
Ans.	1	18	5	4	0

SECTION XVII.

ADDITION OF DECIMALS.

(p. 111.)

6.

73.29
87.047
3005.0106
28.03
29000.005
Ans. 32193.3826

7.

209000.000046
98207.0015
15.08
.0049
Ans. 307222.086446

8.

23000010.
1000.00005
27.000019
7.5
Ans. 23001044.500069

9.

59.059
25000.0025
5.000005
205.05
Ans. 25269.111505

10.

25.000007
145.643
175.89
17.00348
Ans. 363.536487

SECTION XVIII.

SUBTRACTION OF DECIMALS.

(p. 112.)

9.

97.7
27.028
Ans. 70.672

10.

315.0027
115.07
Ans. 199.9327

11.

29004005.

29000.
349200.00024

378200.00024
Ans. 28625804.99976

12.

1000000.
.000001
Ans. 999999.999999

SECTION XXIV.

CIRCULATING DECIMALS.

CASE I.

2. (p. 124.) $.\dot{3} = \frac{3}{9} = \frac{1}{3}$ Ans.
3. $\dot{1}.6\dot{2} = 1.\dot{6}2\dot{1} = 1\frac{621}{999} = 1\frac{23}{37}$ Ans.
4. $.\dot{7}6923\dot{0} = \frac{769230}{999999} = \frac{10}{13}$ Ans.

CASE II.

2. $.5\dot{3} = \frac{5}{10} + \frac{3}{90} = \frac{8}{15}$ Ans.
3. $.5\dot{9}2\dot{5} = \frac{5}{10} + \frac{925}{9990} = \frac{5920}{9990} = \frac{16}{27}$ Ans.
4. $.008\dot{4}9713\dot{3} = \frac{8}{1000} + \frac{497133}{999999000} = \frac{83}{9768}$ Ans.
5. $3\dot{1}.6\dot{2} = 31.621 = 31\frac{621}{999} = 31\frac{23}{37}$ Ans.

CASE III.

2. $3.\dot{6}7\dot{1} = 3.\dot{6}7167167167\dot{1}$; $1.\dot{0}07\dot{1} = 1.\dot{0}0710071007\dot{1}$; $8.\dot{5}\dot{2} = 8.\dot{5}2525252525\dot{2}$; $7.\dot{6}1632\dot{5} = 7.\dot{6}1632561632\dot{5}$ Ans.

3. $1.\dot{5}\dot{2} = 1.52\dot{5}25\dot{2}$; $8.\dot{7}15\dot{6} = 8.71\dot{5}67\dot{1}$; $3.56\dot{7} = 3.56\dot{7}77\dot{7}$; $1.3\dot{7}\dot{8} = 1.3\dot{7}878\dot{7}$ Ans.

4. $.00\dot{0}\dot{7} = .000\dot{7}0707\dot{0}$; $.141\dot{4}1\dot{4} = .141\dot{4}1441\dot{4}$; $887.\dot{1} = 887.111\dot{1}1111\dot{1}$ Ans.

CASE IV.

3. As the denominator 11 cannot be divided by 2, 5, or 10, the decimal is infinite. Then, 11)$\frac{99}{9}$. As two 9's are used, the circulate will consist of two places; thus, $\frac{11)1.00}{.\dot{0}\dot{9}}$ Ans.

4. $\begin{array}{r}253)\underline{9999999999999999999999}\\ 39525691699604743083\end{array}$. As twenty-two 9's are necessary to complete the division without a remainder, so we infer that the repetend will consist of twenty-two places; thus, $\frac{107}{253} = .\dot{4}22924901185770750988\dot{1}$ Ans.

SECTION XXV.

ADDITION OF CIRCULATING DECIMALS

(p. 127.)

2.

$\dot{2}7.5\dot{6} = 27.5\dot{6}75675675675\dot{5}$

$5.\dot{6}3\dot{2} = 5.6\dot{3}26326326326\dot{6}$

$6.\dot{7} = 6.\dot{7}777777777777\dot{7}$

$16.3\dot{5}\dot{6} = 16.3\dot{5}6565656565\dot{6}$

$.7\dot{1} = .7\dot{1}1111111111\dot{1}$

$6.\dot{1}23\dot{4} = 6.1\dot{2}3412341234\dot{1}$

Ans. $63.1\dot{6}9067086888\dot{8}$

3.

$2.7\dot{6}\dot{5} = 2.7\dot{6}56\dot{5}$

$7.1\dot{6}67\dot{4} = 7.1\dot{6}67\dot{4}$

$\dot{3}.67\dot{1} = 3.6\dot{7}13\dot{6}$

$.\dot{7} = .\dot{7}777\dot{7}$

$.\dot{1}72\dot{8} = .1\dot{7}28\dot{1}$

Ans. $14.5\dot{5}43\dot{6}$

4.

$5.\dot{1}634\dot{5} = 5.\dot{1}634516345163451634\dot{5}$

$8.\dot{6}38\dot{1} = 8.\dot{6}381638163816381638\dot{1}$

$3.\dot{7}\dot{5} = 3.\dot{7}575757575757575757\dot{5}$

Ans. $17.\dot{5}591912084737409030\dot{2}$

5.

$\frac{1}{3} = .\dot{3} = .\dot{3}3333\dot{3}$

$\frac{1}{7} = .\dot{1}4285\dot{7} = .\dot{1}4285\dot{7}$

$\frac{1}{9} = .\dot{1} = .\dot{1}1111\dot{1}$

Ans. $.\dot{5}8730\dot{1}$

SECTION XXVI.

SUBTRACTION OF CIRCULATING DECIMALS.

(p. 128.)

2.

$7.\dot{1} = 7.1\dot{1}$
$5.0\dot{2} = 5.0\ddot{2}$

Ans. $2.0\dot{8}$

3.

$31\dot{5}.8\dot{7} = 315.\dot{8}7587587587\dot{5}$
$78.\dot{0}37\dot{8} = 78.\dot{0}3780378037\dot{8}$

Ans. $237.\dot{8}3807209549\dot{7}$

4.

$\frac{2}{9} = .\dot{2} = .\dot{2}2222\dot{2}$
$\frac{1}{7} = .\dot{1}4285\dot{7} = .\dot{1}4285\dot{7}$

Ans. $.\dot{0}7936\dot{5}$

5.

$16.13\dot{4}\dot{7} = 16.13\dot{4}\dot{7}$
$11.08\dot{8}\dot{4} = 11.08\dot{8}\dot{4}$

Ans. $5.04\dot{6}\dot{2}$

6.

$18.167\dot{8} = 18.16\dot{7}\dot{8}$
$3.\dot{2}\dot{7} = 3.27\dot{2}\dot{7}$

Ans. $14.89\dot{5}\dot{1}$

7.

$3.\dot{1}2\dot{3} = 3.\dot{1}2312\dot{3}$
$.\dot{7}\dot{1} = .\dot{7}1717\dot{1}$

Ans. $2.\dot{4}0595\dot{1}$

8.

$\frac{3}{7} = .\dot{4}2857\dot{1} = .\dot{4}2857\dot{1}$
$\frac{2}{11} = .\dot{1}\dot{8} = .\dot{1}8181\dot{8}$

Ans. $.\dot{2}4675\dot{3}$

9.

$\frac{4}{9} = .\dot{4} = .\dot{4}4444\dot{4}$
$\frac{2}{7} = .\dot{2}8571\dot{4} = .\dot{2}8571\dot{4}$

Ans. $.\dot{1}5873\dot{0}$

10.

$\frac{9}{17} = .\dot{5}29411764705882\dot{3}$
$\frac{6}{17} = .\dot{3}52941176470588\dot{2}$

Ans. $.\dot{1}76470588235294\dot{1}$

11.

$$\begin{array}{rl} 5.\dot{1}234\dot{5} & = 5.1\dot{2}34512345123451234512345123451\dot{1} \\ 2.3\dot{5}23456\dot{6} & = 2.3\dot{5}23456523456523456523456523456\dot{6} \\ \hline & \text{Ans. } 2.7\dot{7}11055821666927777988888599994\dot{4} \end{array}$$

SECTION XXVII.

MULTIPLICATION OF CIRCULATING DECIMALS

(p. 129.)

3.

$$\begin{array}{r} 87.32\dot{5}8\dot{6} \\ 437 \\ \hline 61128106 \\ 26197759_{7} \\ 34930346_{34} \\ \hline \text{Ans. } 381.6140\dot{3}3\dot{8} \end{array}$$

4. $3.1\dot{4}\dot{5} = 3.1\frac{45}{99} = 3.1\frac{5}{11} = \frac{348}{110}$; $4.\dot{2}9\dot{7} = 4\frac{297}{999} = 4\frac{11}{37} = \frac{159}{37}$; $\frac{348}{110} \times \frac{159}{37} = \frac{55014}{4070} = 13.5\dot{1}6953\dot{3}$ Ans.

5.

$$\begin{array}{r} .\dot{2}8571\dot{4} \\ 28 \\ \hline 2285714 \\ 571428_{5} \\ \hline \text{Ans. } 8.\dot{0}0000\dot{0} \end{array}$$

6.

$$\begin{array}{r} .461607\dot{1}4285\dot{7} \\ 20 \\ \hline 9)232142\dot{8}5714_{\dot{2}} \\ 4 \\ \hline 0)928571\dot{4}2857\dot{1} \\ 28 \\ \hline 7428571428571 \\ 1857142\dot{8}57142_{8} \\ \hline \text{Ans. } 26.000000\dot{0}0000\dot{0} \end{array}$$

7.

$$\begin{array}{r} .\dot{2}8493150\dot{6} \\ 365 \\ \hline 1424657534 \\ 1709589041_{0} \\ 854794520_{64} \\ \hline \text{Ans. } 104.\dot{0}0000000\dot{0} \end{array}$$

SECTION XXVIII.

DIVISION OF CIRCULATING DECIMALS.

2. (p. 130.) $345.\dot{8} = 345.\frac{8}{9}$; $\dot{6} = \frac{6}{9} = \frac{2}{3}$; $345\frac{8}{9} \div \frac{2}{3} = 518\frac{5}{6} = 518.8\dot{3}$ Ans.

3. $234.\dot{6} = 234\frac{2}{3}$; $\dot{7} = \frac{7}{9}$; $234\frac{2}{3} \div \frac{7}{9} = 301\frac{5}{7} = 301.\dot{7}1428\dot{5}$ Ans.

4. $.\dot{3}\dot{6} = \frac{36}{99} = \frac{12}{33}$; $.2\dot{5} = \frac{2}{10} + \frac{5}{90} = \frac{23}{90}$; $\frac{12}{33} \div \frac{23}{90} = \frac{12}{33} \times \frac{90}{23} = \frac{1080}{759} = 1.\dot{4}22924901185770750988\dot{1}$ Ans.

SECTION XXXI.

PARTIAL PAYMENTS.

(p. 140.)

2.

Principal, ...	$700.00.0
Interest from Feb. 4 to Nov. 28, 9mo. 24da. .	34.30.0
Amount carried forward, ...	$734.30.0

Amount brought forward, ...		$734.30.0
First payment, ...	160.00.0	
Interest from March 18 to Nov. 28, 8mo. 10da.	6.66.6	
Second payment, ...	200.00.0	
Interest from June 24 to Nov. 28, 5mo. 4da.	5.13.3	
Third payment, ...	120.00.0	
Interest from Sept. 11 to Nov. 28, 2mo. 17da.	1.54.0	
Fourth payment, ...	60.00.0	
Interest from Oct. 5 to Nov. 28, 1mo. 23da.	.53.0	
		553.86.9
Balance remaining due Nov. 28, 1834, . . .		$180.43.1

3.

Principal, bearing interest from Aug. 16, 1834,		$600.00.0
Interest from Aug. 16, 1834, to Aug. 1, 1835, 11mo. 15da.		34.50.0
Amount, ...		634.50.0
First payment, Sept. 18, 1834, . . .	136.00.0	
Interest from Sept. 18, 1834, to Aug. 1, 1835, 10mo. 13da.	7.09.4	
Second payment, Dec. 5, 1834, . . .	197.00.0	
Interest from Dec. 5, 1834, to Aug. 1, 1835, 7mo. 26da.	7.74.8	
Amounts carried forward,	$347.84.2	$634.50.0

Amounts brought forward,	$347.84.2	$634.50.0
Third payment, Feb. 11, 1835,	200.00.0	
Interest from Feb. 11, 1835, to Aug. 1, 1835, 5mo. 20da.	5.66.6	
Fourth payment, April 19, 1835,	40.00.0	
Interest from April 19, 1835, to Aug. 1, 1835, 3mo. 12da.	.68.0	
		594.18.8
Balance remaining due Aug. 1, 1835,		$40.31.2

5.

Principal, on interest from June 17, 1829,	$769.87.0
Interest from June 17, 1829, to March 1, 1830, 8mo. 14da.	32.59.1
Amount,	802.46.1
First payment, March 1, 1830,	75.50.0
New principal, bearing interest from Mar. 1, 1830,	726.96.1
Interest from March 1, 1830, to June 11, 1831, 15mo. 10d.	55.73.3
Amount,	782.69.4
Second payment, June 11, 1831,	165.00.0
New principal, bearing interest from June 11, 1831,	617.69.4
Interest from June 11, 1831, to Sept. 15, 1831, 3mo. 4da.	9.67.7
Amount,	627.37.1
Third payment, Sept. 15, 1831,	161.00.0
New principal, bearing interest from Sept. 15, 1831, carried forward,	$466.37.1

Amount brought forward,		$466.37.1
Interest from Sept. 15, 1831, to Jan. 21, 1832, 4mo. 6da.		9.79.3
Amount,		476.16.4
Fourth payment, January 21, 1832,		47.25.0
New principal, bearing interest from Jan. 21, 1832,		428.91.4
Interest from Jan. 21, 1832, to Dec. 6, 1833, 22mo. 15da.		48.25.2
Amount,		477.16.6
Fifth payment, less than interest, March 5, 1833,	12.17	
Sixth payment, more than interest, Dec. 6, 1833,	98.00	110.17.0
New principal, bearing interest from Dec. 6, 1833,		366.99.6
Interest from Dec. 6, 1833, to July 7, 1834, 7mo.1d.		12.90.6
Amount,		379.90.2
Seventh payment, July 7, 1834,		169.00.0
New principal, bearing interest from July 7, 1834,		210.90.2
Interest from July 7, 1834, to Sept. 25, 1835, 14mo. 18da.		15.39.5
Balance due Sept. 25, 1835,		$226.29.7

6.

Principal, on interest from April 30, 1831,	$300.00.0
Interest from April 30, 1831, to June 27, 1832, 13mo. 27da.	20.85.0
Amount carried forward,	$320.85.0

Amount brought forward, . . .	$320.85.0
First payment, June 27, 1832, ...	150.00.0
New principal, bearing interest from June 27, 1832,	170.85.0
Interest from June 27, 1832, to Dec. 9, 1832, 5mo. 12da.	4.61.2
Amount, ...	175.46.2
Second payment, Dec. 9, 1832, ...	150.00.0
New principal, bearing interest from Dec. 9, 1832,	25.46.2
Interest from Dec. 9, 1832, to Oct. 9, 1833, 10mo.	1.27.3
Balance due Oct. 9, 1833, ...	$26.73.5

7.

Principal, on interest from Feb. 11, 1832, . . .		$54.18.0
Interest from Feb. 11, 1832, to July 11, 1833, 17mo.		4.60.5
Amount, ...		58.78.5
First payment, July 11, 1833, ...		12.25.0
Principal, bearing interest from July 11, 1833, .		46.53.5
Interest from July 11, 1833, to Aug. 21, 1835, 25mo. 10da.		5.89.4
Amount, ...		52.42.9
Second payment, less than interest, Aug. 15, 1834, ...	2.10.0	
Third payment, less than interest, July 9, 1835, ...	3.12.0	
Amounts carried forward, .	$5.22.0	$52.42.9

Amounts brought forward,	$5.22.0	$52.42.9
Fourth payment, more than interest, Aug. 21, 1835,	37.18.0	
		42.40.0
Principal, bearing interest from Aug. 21, 1835,		10.02.9
Interest from Aug. 21, 1835, to Dec. 17, 1835, 3mo. 26da.		.19.3
Balance due, Dec. 17, 1835,		$10.22.2

8.

Principal, on interest from Jan. 7, 1831,		$1728.00.0
Interest from Jan. 7, 1831, to Feb. 9, 1832, 13mo. 2da.		112.89.6
Amount,		1840.89.6
First payment, Feb. 9, 1832,		760.28.5
Principal, bearing interest from Feb. 9, 1832,		1080.61.1
Interest from Feb. 9, 1832, to Dec. 28, 1833, 22mo. 19da.		122.28.9
Amount,		1202.90.0
Second payment, less than interest, Mar. 5, 1833,	68.50	
Third payment, more than interest, Dec. 28, 1833,	876.28	
		944.78.0
Principal, bearing interest from Dec. 28, 1833,		258.12.0
Interest from Dec. 28, 1833, to July 17, 1834, 6mo. 19da.		8.56.0
Amount carried forward,		$266.68.0

Amount brought forward,		$266.68.0
Fourth payment, July 17, 1834,		60.00.0
Principal, bearing interest from July 17, 1834,		206.68.0
Interest from July 17, 1834, to Oct. 1, 1834, 2mo. 14da.		2.54.9
Balance due Oct. 1, 1834,		$209.22.9

9.

Principal, on interest from May 7, 1829,		$500.00.0
Interest from May 7, 1829, to June 29, 1830, 13mo. 22da.		34.33.3
Amount,		534.33.3
First payment, June 29, 1830,		100.00.0
New principal, carrying interest from June 29, 1830,		434.33.3
Interest from June 29, 1830, to Dec. 5, 1831, 17mo. 6da.		37.35.2
Amount,		471.68.5
Second payment, Dec. 5, 1831,		100.00.0
New principal carrying interest from Dec. 5, 1831,		371.68.5
Interest from Dec. 5, 1831, to July 4, 1833, 18mo. 29da.		35.24.8
Amount,		406.93.3
Third payment, March 12, 1832, less than interest,	5.00	
Fourth payment, July 4, 1833, more than interest,	95.00	100.00.0
New principal, carrying interest from July 4, 1833, carried forward,		$306.93.3

Amount brought forward,	$306.93.3
Interest from July 4, 1833, to Dec. 1, 1834, 16mo. 27da.	25.93.6
Amount,	332.86.9
Fifth payment, Dec. 1, 1834,	200.00.0
New principal, carrying interest from Dec. 1, 1834,	132.86.9
Interest from Dec. 1, 1834, to Jan. 1, 1836, 13mo.	8.63.6
Balance due Jan. 1, 1836,	$141.50.5

10.

Principal, on interest from March 19, 1831,	$89.75.0
Interest from March 19, 1831, to Dec. 6, 1831, 8mo. 17da.	3.84.4
Amount,	93.59.4
First payment, Dec. 6, 1831,	12.12.0
New principal, bearing interest from Dec. 6, 1831,	81.47.4
Interest from Dec. 6, 1831, to Feb. 17, 1832, 2mo. 11da.	.96.4
Amount,	82.43.8
Second payment, Feb. 17, 1832,	12.12.0
New principal, bearing interest from Feb. 17, 1832,	70.31.8
Interest from Feb. 17, 1832, to Jan. 1, 1835, 34mo. 14da.	12.11.8
Amount carried forward,	$82.43.6

Amount brought forward,		$82.43.6
Third payment, March 19, 1833, less than interest,	3.16	
Fourth payment, Dec. 28, 1834, less than interest,	2.18	
Fifth payment, Jan. 1, 1835, more than interest,	25.25	
		30.59.0
New principal, bearing interest from Jan. 1, 1835,		51.84.6
Interest from Jan. 1, 1835, to March 11, 1835, 2mo. 10da.		.60.4
Amount,		52.45.0
Sixth payment, March 11, 1835,		31.18.0
New principal, bearing interest from Mar. 11, 1835,		21.27.0
Interest from March 11, 1835, to July 17, 1835, 4mo. 6da.		.44.6
Amount,		21.71.6
Seventh payment, July 17, 1835,		5.18.0
New principal, bearing interest from July 17, 1835,		16.53.6
Interest from July 17, 1835, to Sept. 1, 1835, 1mo. 14da.		.12.1
Amount,		16.65.7
Eighth payment, Sept. 1, 1835,		6.29.0
New principal, bearing interest from Sept. 1, 1835,		10.36.7
Interest from Sept. 1, 1835, to Dec. 29, 1835, 3mo. 28da.		.20.3
Balance due Dec. 29, 1835,		$10.57.0

SECTION XXXII.

MISCELLANEOUS PROBLEMS.

2. (p. 146.) $\$120 \times .01 = 1.20 \times 1\frac{1}{3} = 1.60$; $133.20 - 120 = 13.20 \div 1.60 = 8\frac{1}{4}$ per cent. Ans.

3. $\$280 \times .01 = 2.80 \times 6\frac{1}{2} = 18.20$; $411.95 - 280 = 131.95 \div 18.20 = 7\frac{1}{4}$ per cent. Ans.

5. $\$120 \times .08\frac{1}{4} = 9.90$; $133.20 - 120 = 13.20 \div 9.90 = 1\frac{1}{3}$y. $= 16$ months, Ans.

6. $\$280 \times .07\frac{1}{4} = 20.30$; $411.95 - 280 = 131.95 \div 20.30 = 6\frac{1}{2}$ years, Ans.

8. $\$1.00 \times .08\frac{1}{4} = .0825 \times 1\frac{1}{3} = .11$; $13.20 \div .11 = \$120.00$ Ans.

9. $.07\frac{1}{4} = .0725 \times 6\frac{1}{2} = .47125 + 1 = 1.47125$; $411.95 \div 1.47125 = \$280.00$.

MISCELLANEOUS QUESTIONS.

1. (p. 148.) $\$1728 \times .15 = 259.20$; $1728 - 260.20 = \$1468.80$ Ans.

2. $\$15000 \times .12\frac{1}{2} = 1875$; $15000 - 1875 = \$13125 \times .25 = 3281.25$; $13125 - 3281.25 = \$9843.75$ Ans.

3. $\$560 \times .12\frac{1}{2} = \$70.00$; $\$400 \times .18 = \$72.00 - 70.00 = \$2.00$ Ans.

4. $.12 + .20 + .40 = .72$; $\$1890 \times .72 = \$1360.80$; $1890 - 1360.80 = 529.20 \div 2646 = .20$ Ans.

SECTION XXXIII.

DISCOUNT.

2. (p. 148.) \$1.12 amount of \$1, for the given time; $\$117.60 \div 1.12 = \$105.00$ Ans.

3. \$1.07 amount of \$1, for the given time; \$802.50 ÷ 1.07 = \$750; \$802.50 — 750 = \$52.50 Ans.

4. \$1.205 amount of \$1, for 3y. 5mo.; \$769.60 ÷ 1.205 = $\$638.67.2\frac{48}{241}$ Ans.

5. $\$1.46.8\frac{1}{3}$ amount of \$1, for 7y. 9mo. 20da.; \$986.40 ÷ $1.46.8\frac{1}{3} = \$671.78.2\frac{58}{881}$ Ans.

6. $\frac{16}{16}$ = amount, $\frac{15}{16}$ = principal, $\frac{1}{16}$ = discount. Therefore, 32 × 16 = 512 ÷ 15 = $34\frac{2}{15}$qts. Ans.

7. \$1.26875 amount of \$1, for 3y. 7mo.; \$678.75 ÷ 1.26875 = $\$534.97.5\frac{75}{203}$ Ans.

8. $\$1.091\frac{5}{6}$ amount of \$1, for 18mo. 11da.; \$1000 ÷ $1.091\frac{5}{6} = \$915.89\frac{461}{6551}$ Ans.

9. $\$1.017\frac{2}{3}$ amount of \$1, for 3mo. 16da.; \$715.50 ÷ $1.017\frac{2}{3} = \$703.07.8\frac{2866}{3053}$ Ans.

10. \$1.054 amount of \$1, for 10mo. 24da.; \$914.75 ÷ 1.054 = $\$867.88.4\frac{132}{527}$ Ans.

11. $\$1.056\frac{5}{6}$ amount of \$1, for 11mo. 11da.; \$79.87 ÷ $1.056\frac{5}{6}$ = \$75.57.4 +.
   $\$1.111\frac{1}{6}$ amount of \$1, for 22mo. 7da.; \$87.75 ÷ $1.111\frac{1}{6}$ = \$78.97.1 + \$75.57.4 = \$154.54.5 + Ans.

12. $\$1.250\frac{1}{2}$ amount of \$1, for 50mo. 3da.; \$1728 ÷ $1.250\frac{1}{2} = \$1381.84.7\frac{653}{2501}$ Ans.

13. \$5.00 × 1.10 = \$5.50; \$1.00 — .10 = .90; 5.50 ÷ .90 = $\$6.11\frac{1}{9}$ Ans.

14. \$1.118 amount of \$1, for 23mo. 18da.; \$365.87 ÷ 1.118 = \$327.25.4 +.
   \$1.1525 amount of \$1, for 30mo. 15da.; \$161.15 ÷ 1.1525 = \$139.82.6 +.
   $\$1.028\frac{2}{3}$ amount of \$1, for 5mo. 22da.; \$112.50 ÷ $1.028\frac{2}{3}$ = \$109.36.4 +.
   \$1.258 amount of \$1, for 51mo. 18da.; \$96.81 ÷ 1.258 = \$76.95.5 +.
   \$327.25.4 + \$139.82.6 + \$109.36.4 + \$76.95.5 + = \$653.40 + Ans

15. First find the interest on the shoes from the time they become due until the time of settlement; add this interest to the value of the shoes.

$865 × 1.068$\frac{5}{6}$ = $924.54; $386.27 × 1.063$\frac{2}{3}$ = $410.86.

$769.25 × 1.056$\frac{2}{3}$ = $812.84; $183.75 × 1.047$\frac{1}{3}$ = $192.44.

$396.81 × 1.043$\frac{1}{3}$ = 414.00.

$924.54 + $410.86 + $812.84 + $192.44 + $414.00 = $2754.68 amount of the shoes.

Then find the amount of the sums A B has received, from the time when received to the time of settlement.

$1000 × 1.061$\frac{1}{3}$ = 1061.33; $375.25 × 1.053$\frac{1}{3}$ = $395.26.

$681.29 × 1.049 = $714.67; $100 × 1.045$\frac{1}{6}$ = $104.51.

$275.28 × 1.029$\frac{1}{2}$ = $283.40.

$1061.33 + $395.26 + $714.67 + $104.51 + $283.40 = $2559.17 amount of the sums received; $2754.68 — $2559.17 = $195.51 Ans.

SECTION XXXIV.

COMMISSION AND BROKERAGE.

1. (p. 150.) $18768 × .01$\frac{3}{4}$ = $328.44 Ans.
2. $896 × .02 = $17.92 Ans.
3. 395£. 15s. 5d. × .02$\frac{1}{4}$ = 8£. 18s. 1$\frac{88}{400}$d. Ans.
4. $1976 × $\frac{100}{104}$ = $1900 Ans.
5. $102.50 × 90 = $9225 ÷ 2$\frac{1}{2}$ = $3690 the sum remitted; $3690 — $90 = $3600 ÷ 95 = 37T. 17cwt. 3qr. 16$\frac{4}{19}$lb. Ans.

SECTION XXXV.

STOCKS.

2. (p. 151.) $\$100 \times 15 = \$1500 \times 1.13 = \$1695$ Ans.
3. $\$100 \times 12 = \$1200 \times 1.15 = \$1380.00$ Ans.
4. 1058£. 12s. $\times\ 1.15\frac{3}{4} =$ 1225£. 6s. $7\frac{2}{25}$d. Ans.
5. $\$100 \times 30 = \$3000 \times 1.08\frac{3}{4} = \$3262.50$ Ans.
6. $\$100 \times 10 = \$1000 \times .85 = \$850$ Ans.
7. $\$100 \times 5 = \$500 \times 1.07 = \$535$ Ans.

SECTION XXXVI.

INSURANCE AND POLICIES.

1. (p. 152.) $\$896 \times .12 = \$107.52$ Ans.
2. $\$850 \times .18\frac{1}{2} = \$157.25$ Ans.
3. $\$9870 \times .07 = \$690.90$ Ans.
4. $\$1.00 - .15 = .85$; $\$1728 \times .85 = \$1468.80$ Ans.
5. $\$1.00 - .10 = .90$; $\$2475 \div .90 = \$2750$ Ans.
6. $\$1.00 - .12\frac{1}{2} = .87\frac{1}{2}$; $\$26250 \div .87\frac{1}{2} = \$30.000$ Ans.
7. $\$3600 \times .60 = \$2160$ Ans.
8. $\$1.00 - .10 = .90$; $\$600 \times .90 = \$540$ Ans.
9. $\$1.00 - .03 = .97$; $\$1000 \div .97 = \$1030.92.7 +$.
   $\$1.00 - .05 = .95$; $\$1030.92.7 \div .95 = \$1085.18.7 +$.
   $\$1.00 - .06 = .94$; $\$1085.18.7 \div .94 = \$1154.45.4 +$.
   $\$1.00 - .07 = .93$; $\$1154.45.4 \div .93 = \$1241.34.8 +$.
   Ans.

SECTION XXXVII.

BANKING.

1. (p. 153.) $\$476 \times .005\frac{1}{2} = \$2.61.8$ Ans.
2. $\$1000 \times .010\frac{1}{2} = \$10.50$ Ans.
3. $\$7800 \times .015\frac{1}{2} = \$120.90$ Ans.
4. $\$8000 \times .010\frac{1}{2} = \$84.00$ Ans.
5. $\$760 \times .025\frac{1}{2} = \$19.38$; $\$760 - \$19.38 = \$740.62$ Ans.
6. $\$1728 \times .15\frac{1}{2} = \$26.78.4$; $\$1728 - \$26.78.4 = \$1701.21.6$ Ans.
7. $\$7860 \times .030\frac{1}{2} = \$239.73$; $\$7860 - \$239.73 = \$7620.27$ Ans.
8. $\$3.50 \times 450 = \$1575$ price for which the fish were bought.

   $\$4.00 \times 450 = \$1800$ price for which they were sold.

   $\$1800 \times .030\frac{1}{2} = \$54.90$; $\$1800 - \$54.90 = \$1745.10$ received at the bank.

   $\$1745.10 - \$1575 = \$170.10$ gain on the fish, Ans.

SECTION XXXVIII.

BARTER.

1. (p. 154.) $760 \times 8 = 6080 \div 12\frac{1}{2} = 486\frac{2}{5}$lb. Ans.
2. $760 \times 62\frac{1}{2} = 47500 \div 17 = 2794\frac{2}{17}$lb. Ans.
3. $3 \times 63 \times 1.10 = 20790 \div 126 = \$1.65$. Ans.
4. $12 \times 112 \times 8 = 10752 \div 196 = 54\frac{6}{7}$lb. Ans.
5. 17cwt. 3qr. 4lb. $= 1992$; $41 \times 6.70 = \$274.70 - \$80 = \$186.70 \div 1992 = \$0.09\frac{371}{996}$ Ans.

6. B's tea cost him 40 cents per lb., and he sells it for 50 cents per lb.; his gain, therefore, is $\frac{1}{4}$ of the principal, or 25 per cent. A's gain is 4 cents on the lb., and his gain, to be in proportion to B's, must be $\frac{1}{4}$ of the principal; A's sugar, therefore, will be $4 \times 4 = 16$ cents per lb. Ans.

7. $15 \times 63 \times 1.25 = \$1181.25 \div 25 = 47\frac{1}{4}$M. staves, Ans.

8. As Q bought his oats for 35 cents per bushel, and sold them at 50 cents, his gain per cent. is $50 - 35 = 15$; $\frac{15}{35} = \frac{3}{7}$ of the cost. If, therefore, we add $\frac{3}{7}$ of the cost of Z's flour to \$5.00, we have his bartering price; thus, $\frac{3}{7}$ of $\$5.00 = \$2.14\frac{2}{7}$, $\$2.14\frac{2}{7} + \$5.00 = \$7.14\frac{2}{7}$ per barrel; and $670 \times .50 = \$335 \div \$7.14\frac{2}{7} = 46\frac{9}{10}$ barrels of flour, Ans.

SECTION XXXIX.

COMPOUND INTEREST.

(p. 155.)

13.

Principal, bearing interest from Jan. 1, 1830,		$500.00.0
Compound interest on $500 from Jan. 1, 1830, to Sept. 1, 1834, 4y. 8mo.		156.48.7
Amount of the principal to Sept. 1, 1834,		656.48.7
First payment, July 16, 1830,	200.00.0	
Compound interest from July 16, 1830, to Sept. 1, 1834, 4y. 1mo. 15da.	54.38.8	
Second payment, Aug. 21, 1831,	200.00.0	
Compound interest from Aug. 21, 1831, to Sept. 1, 1834, 3y. 0mo. 10da.	38.59.9	
Amounts carried forward,	$492.98.7	$656.48.7

Amounts brought forward,	$492.98.7	$656.48.7
Third payment, Dec. 1, 1832,	100.00.0	
Compound interest from Dec. 1, 1832, to Sept. 1, 1834, 1y. 9mo.	10.77.0	
Amount of the endorsements,		603.75.7
Balance due Sept. 1, 1834,		$52.73.0

14.

Principal, bearing interest from March 25, 1833,		$100.00.0
Interest for 1y. 5mo.		8.65.0
Amount of the principal to Aug. 25, 1835,		108.65.0
First payment, June 11, 1834,	50.00.0	
Compound interest from June 11, 1834, to Aug. 25, 1835, 14mo. 14da.	3.65.3	
Second payment, Sept. 25, 1834,	50.00.0	
Compound interest from Sept. 25, 1834, to Aug. 25, 1835, 11mo.	2.75.0	
Amount of the endorsements,		106.40.3
Balance due Aug. 25, 1835,		$2.24.7

SECTION XL.

PRACTICE.

(p. 160.)

6.

6d. = $\frac{1}{2}$) 387s. 0d. = price at 1s.
3d. = $\frac{1}{2}$) 193 6 = price at 6d.
96 9 = price at 3d.
20) 290 3
Ans. 14£. 10s. 3d. = price at 9d.

7.

2s. 6d. = $\frac{1}{8}$) 490£. 0s. 0d. = value at 1£. per lb.

Ans. 61£. 5s. 0d. = value at 2s. 6d.

8.

4s. = $\frac{1}{5}$) 384£. value at 1£. per yd.

6d. = $\frac{1}{8}$) 76 16 0 = value at 4s.

3d. = $\frac{1}{2}$) 9 12 0 = value at 6d.

4 16 0 = value at 3d.

Ans. 91£. 4s. 0d. = value at 4s. 9d.

9.

10s. = $\frac{1}{2}$) 714£. value at 1£. per yd.

5s. = $\frac{1}{2}$) 357 00 = value at 10s.

6d. = $\frac{1}{10}$) 178 10 = value at 5s.

17 17 = value at 6d.

Ans. 553£. 7s. = value at 15s. 6d

10.

2qr. = $\frac{1}{2}$) \$2.50 = value of 1cwt.

16

40.00 = value of 16cwt.

1qr. = $\frac{1}{2}$) 1.25 = value of 2qr.

7lb. = $\frac{1}{4}$) .62.5 = value of 1qr.

2lb. = $\frac{1}{14}$) .15.62 = value of 7lb.

1lb. = $\frac{1}{2}$) 4.49 = value of 2lb.

2.24 = value of 1lb.

Ans. 42.09.8 + = value of 16cwt. 3qr. 10lb.

11.

1qr. = ¼)	$14	= value of 1cwt.
	27	
	378	= value of 27cwt.
14lb. = ½)	3.50	= value of 1qr.
7lb. = ½)	1.75	= value of 14lb.
	87½	= value of 7lb.
Ans.	$384.12½	= value of 27cwt. 1qr. 21lb

12.

10cwt. = ½)	$24.60	= value of 1 ton.
	7	
	172.20	= value of 7 tons.
2cwt. = ⅕)	12.30	= value of 10cwt.
1cwt. = ½)	2.46	= value of 2cwt.
2qr. = ½)	1.23	= value of 1cwt.
7lb. = ⅛)	.61.5	= value of 2qr.
	7.6⅞	= value of 7lb.
Ans.	$188.88.1⅞	= value of 7T. 13cwt. 2qr 7lb

13.

2R. = ½)	$80.50	= value of 1 acre.
	25	
	2012.50	= value of 25 acres.
20rd. = ¼)	40.25	= value of 2 roods.
10rd. = ½)	10.06.2½	= value of 20 rods.
5rd. = ½)	5.03.1¼	= value of 10 rods.
	2.51.5⅝	= value of 5 rods.
Ans.	$2070.35.9⅜	= value of 25A. 2R. 35rd.

14.

10rd. = $\frac{1}{16}$) \$32.32 = value of 1A.
51
3232
16160
\$1648.32 = value of 51A.
5rd. = $\frac{1}{2}$) 2.02 = value of 10rd.
1.01 = value of 5rd.
Ans. \$1651.35 = value of 51A. 0R. 15rd.

15.

2qr. = $\frac{1}{2}$) \$5.60 = value of 1yd.
7
39.20 = value of 7yd.
1qr. = $\frac{1}{2}$) 2.80 = value of 2qr.
2na. = $\frac{1}{2}$) 1.40 = value of 1qr.
.70 = value of 2na.
Ans. \$44.10 = value of 7yd. 3qr. 2na.

16.

10 = $\frac{1}{10}$) \$6780 = principal.
$2\frac{1}{2}$ = $\frac{1}{4}$) 678 = 10 per cent.
169.50 = $2\frac{1}{2}$ per cent.
Ans. \$847.50 = $12\frac{1}{2}$ per cent.

17.

5 per cent. $= \frac{1}{20}$)	\$1728.00	= principal.
1 per cent. $= \frac{1}{5}$)	86.40	= interest at 5 per cent.
	17.28	= interest at 1 per cent.
6 months $= \frac{1}{2}$)	103.68	= interest at 6 per cent.
	5	
	518.40	= interest for 5 years.
1 month $= \frac{1}{6}$)	51.84	= interest for 6 months.
15 days $= \frac{1}{2}$)	8.64	= interest for 1 month.
5 days $= \frac{1}{3}$)	4.32	= interest for 15 days.
	1.44	= interest for 5 days.
Ans.	\$584.64	= interest for 5y. 7mo. 20da.

18.

10cwt. $= \frac{1}{2}$)	19£.	19s.	$11\frac{3}{4}$d.	= value of 1 ton.
			19	
	379	19	$7\frac{1}{4}$	= value of 19 tons.
5cwt. $= \frac{1}{2}$)	9	19	$11\frac{7}{8}$	= value of 10cwt.
4cwt. $= \frac{1}{5}$)	4	19	$11\frac{15}{16}$	= value of 5cwt.
2qr. $= \frac{1}{8}$)	3	19	$11\frac{19}{20}$	= value of 4cwt.
1qr. $= \frac{1}{2}$)	0	9	$11\frac{159}{160}$	= value of 2qr.
14lb. $= \frac{1}{2}$)	0	4	$11\frac{319}{320}$	= value of 1qr.
7lb. $= \frac{1}{2}$)	0	2	$5\frac{639}{640}$	= value of 14lb.
4lb. $= \frac{1}{7}$)	0	1	$2\frac{1279}{1280}$	= value of 7lb.
2lb. $= \frac{1}{2}$)	0	0	$8\frac{1279}{2240}$	= value of 4lb.
$\frac{1}{2}$lb. $= \frac{1}{4}$)	0	0	$4\frac{1279}{4480}$	= value of 2lb.
	0	0	$1\frac{1279}{17920}$	= value of $\frac{1}{2}$lb.
Ans.	399£.	19s.	$5\frac{16641}{17920}$d.	= value of 19T. 19cwt. 3qr. $27\frac{1}{2}$lb.

SECTION XLI.

EQUATION OF PAYMENTS.

2. (p. 162.) \$50 to be paid in 2 months, is the same as \$100 in 1 month; \$150 in 8 months, is the same as \$100 in 12 months. The equated time, then, will be, as if \$100 were to be paid in 1 month, \$100 in 5 months, and \$100 in 12 months; that is, \$100 in $1+5+12=18$ months; consequently \$300 in $\frac{1}{3}$ of 18 months $=$ 6 months, Ans.

OPERATION.

$$\begin{array}{rr}
\$\ 50 \times 2 = & 100 \\
\$100 \times 5 = & 500 \\
\$150 \times 8 = & 1200 \\
\hline
\$300 \quad 300\,) & 1800\,(\,6 \text{ months Ans.} \\
 & 1800 \\
\hline
\end{array}$$

3. \$200 to be paid in 3 months, is the same as \$100 in 6 months; and \$300 in 5 months, is the same as \$100 in 15 months; and \$500 in 10 months, is the same as \$100 in 50 months. If, then, \$100 is to be paid in $6+15+50=71$ months, \$1000 should be paid in $\frac{1}{10}$ of 71 months $=\frac{71}{10}=$ 7mo. 3da. Ans.

OPERATION.

$$\begin{array}{rr}
\$\ 200 \times \ 3 = & 600 \\
\$\ 300 \times \ 5 = & 1500 \\
\$\ 500 \times 10 = & 5000 \\
\hline
\$1000 \quad 1000\,) & 7100\,(7\tfrac{1}{10} \text{ months, Ans.} \\
 & 7000 \\
\hline
 & 100
\end{array}$$

4. \$50 for 4 months, is \$1 for 200 months; and \$100 for 8 months, is \$1 for 800 months. If, then, \$1 is to be paid in $200 + 800 = 1000$ months, \$150 should be paid in $\frac{1}{150}$ of 1000 months $= \frac{1000}{150} = 6\frac{2}{3}$ months. It appears, then, that \$150 is paid $6\frac{2}{3}$ months before it is due. The question now is, In how much *less* than 10 months should the \$250 be paid? As we have before shown that \$1 is to be paid in 1000 months, \$250 should be paid in $\frac{1}{250}$ of 1000 months $= 1000 \div 250 = 4$ months. We find, then, that the time in which B is to pay A, is 4 months *less* than it otherwise would have been, on account of A's making present payment. We therefore subtract 4 months from 10, and find the true answer; thus, $10 - 4 = 6$ months, Ans.

OPERATION.

$$\begin{array}{rcl} 50 \times 4 = & 200 & \\ 100 \times 8 = & 800 & \\ \hline 150 \quad 150\,) & 1000 & (\,6\frac{2}{3} \text{ months.} \\ & 1000 & \\ \hline \end{array}$$

$$150 \times 6\frac{2}{3} = 1000 \div 250 = 4 \text{ months};$$
$$10 - 4 = 6 \text{ months, Ans.}$$

5. As $\frac{1}{2}$ of \$144 is paid 7 months before it is due, it is evident the remainder, \$72, is payable in 14 months; but \$48 of this sum is paid in 4 months, which is 10 months before it is due. There will then remain \$24 unpaid; and the question will be, How long may \$24 be kept to pay the interest of \$72 for 10 months? \$72 for 10 months, is the same as \$1 for 720 months; and \$1 for 720 months, is the same as \$24 for $\frac{1}{24}$ of 720 months $= 720 \div 24 = 30$ months. To the 30 months we add the 4 months, and we have the whole equated time; thus, $30 + 4 = 34$ months $=$ 2y. 10mo. Ans

OPERATION.

$$\begin{array}{lll}
\underline{144 \times 7} = 1008 & & \\
\;\;72 & & \\
\;\;\underline{48} \times 4 = \;\;\underline{192} & & \\
\underline{120} & 24\,)\,816\,(& 34\text{mo.} = 2\text{y. } 10\text{mo. Ans.} \\
\;\;24 & \quad\;\;\underline{72} & \\
 & \quad\;\;\;96 & \\
 & \quad\;\;\;\underline{96} &
\end{array}$$

6. \$133⅓ in 2 months, is the same as \$1 in 266⅔ months; and \$266⅔ in 3 months, is the same as \$1 in 800 months; \$400 in 6 months, is the same as \$1 in 2400 months; and \$1 in 266⅔ + 800 + 2400 = 3466⅔ months, is the same as \$800 for $\frac{1}{800}$ of 3466⅔ months = 3466⅔ ÷ 800 = 4⅓ months. And if ½ of \$800 be paid down, the remainder, \$400, may be kept twice 4⅓ months = 8⅔ months, Ans.

OPERATION.

$$\begin{array}{lll}
\$133\tfrac{1}{3} \times 2 = & 266\tfrac{2}{3} & \\
\;\;266\tfrac{2}{3} \times 3 = & 800 & \\
\;\;\underline{400} \;\;\times 6 = & \underline{2400} & \\
\;\;800 \quad 400\,) & 3466\tfrac{2}{3} & (\,8 \text{ months.} \\
\;\;\underline{400} & \underline{3200} & \\
\;\;400 & \;\;266\tfrac{2}{3} & \\
 & \;\;\;\;\underline{30} & \\
\quad\quad 400\,) & 8000 & (\,20 \text{ days.} \\
 & \underline{800} & \\
 & \quad\;\;0 &
\end{array}$$

8. The various parcels of goods will become due as follows:—

May 1,	a bill	amounting	to	$600.
July 7,	"	"	"	$370.
July 15,	"	"	"	$560.
Oct. 20,	"	"	"	$420.

OPERATION.

$$\begin{array}{rrl}
\$600 \times & 0 = & \\
370 \times & 67 = & 24790 \\
560 \times & 75 = & 42000 \\
\underline{420} \times & 172 = & \underline{72240} \\
\$1950 & 1950\,) & 139030\,(\,71\tfrac{58}{195}\ \text{Ans.} \\
 & & \underline{13650} \\
 & & 2530 \\
 & & \underline{1950} \\
 & & 580
\end{array}$$

The medium time of payment will therefore be $71\frac{58}{195}$ days, that is, 72 days from May 1, which will be July 11.

NOTE.—Merchants generally prefer the following method:

$$\begin{array}{rl}
\$600 \times 0 & \\
370 \times 2\tfrac{6}{30} = & 814 \\
560 \times 2\tfrac{14}{30} = & 1381\tfrac{1}{3} \\
\underline{420} \times 5\tfrac{18}{30} = & \underline{2366} \\
\$1950 \quad 1950\,) & 4561\tfrac{1}{3}\,(\,2\text{mo. } 10\tfrac{34}{195}\text{da. Ans.}
\end{array}$$

SECTION XLII.

AMERICAN DUTIES.

(p. 169.)

6.

1276
1280
1178
1378
1570
1338
8020
42 draft.
7978
957 tare.
7021 net.
2½
14042
35105
$175.52.5 Ans.

7.

986£.
40
9) 39440
$4382.22
.44
1752888
1752888
$1928.17.6.8 Ans:

8.

176
145
128
148
150
747
20 wants.
727
15 leakage
712
.15
3560
712
$106.80 Ans

9.

270 tons.
30
$8100 Ans.

10.

7890 × .01¾ = 138.07
7890 — .138 = 7752
7752 × .04 = $310.08 Ans.

11.

$450 \times 10 = 4500$lb.
40 draft.
4460
$357 = 8$ per cent.
4103
.04
$164.12 Ans.

SECTION XLIV.

PROPORTION.

32. (p. 175.) **BY ANALYSIS.**

If $\frac{1}{2}$yd. cost $\frac{3}{5}$£., 1yd. will cost $\frac{6}{5}$£., and $\frac{1}{8}$yd. will cost $\frac{1}{8} \times \frac{6}{5} = \frac{6}{40} = \frac{3}{20}$£.; and $\frac{7}{8}$ will cost 7 times $\frac{3}{20}$£. $= \frac{7}{1} \times \frac{3}{20} = \frac{21}{20} = 1$£. 1s. 0d. Ans.

STATEMENT.

$\frac{1}{2}$yd. : $\frac{7}{8}$yd. : : $\frac{3}{5}$£. $= \frac{2}{1} \times \frac{7}{8} \times \frac{3}{5} = \frac{42}{40} = \frac{21}{20}$£. $= 1$£. 1s. 0d. Ans.

33. **BY ANALYSIS.**

In $4\frac{1}{2}$yd. are 9 halves; in $13\frac{1}{2}$yd. are 27 halves. If, therefore, 9 halves cost $9.75, 1 half will cost $\frac{1}{9}$ of \$9.75 $= \frac{975}{9}$, and 27 halves will cost 27 times $\frac{975}{9} = \frac{27}{1} \times \frac{975}{9} =$ \$29.25 Ans.

FORM OF STATEMENT.

$4\frac{1}{2}$yd. : $13\frac{1}{2}$yd. : : $9.75 : $29.25 Ans.

OPERATION.

$\frac{2}{9} \times \frac{27}{2} \times \frac{975}{1} = \frac{52650}{18} =$ \$29.25 Ans.

34. BY ANALYSIS.

If it be 1in. wide, the length must be 144in.; and if it be $2\frac{1}{2}$in., it will be $144 \div 2\frac{1}{2} = 57\frac{3}{5}$in. Ans.

FORM OF STATEMENT.

$2\frac{1}{2}$in. : 1in. : : 144in. : $57\frac{3}{5}$in. Ans.

35. BY ANALYSIS.

If $\frac{7}{16}$ cost 51£., $\frac{1}{16}$ will cost $\frac{51}{7}$; and $\frac{16}{16}$, or the whole ship, will cost 16 times $\frac{51}{7}$£. $= \frac{16}{1} \times \frac{51}{7} = \frac{816}{7}$£.; $\frac{1}{32}$ of $\frac{816}{7} = \frac{1}{32} \times \frac{816}{7} = \frac{816}{224} = \frac{51}{14}$; and $\frac{3}{32}$ will cost 3 times $\frac{51}{14} = \frac{3}{1} \times \frac{51}{14} = \frac{153}{14}$£. = 10£. 18s. $6\frac{6}{7}$d. Ans.

FORM OF STATEMENT.

$\frac{7}{16} : \frac{3}{32}$: : 51£. : 10£. 18s. $6\frac{6}{7}$d. Ans.

OPERATION.

$\frac{16}{7} \times \frac{3}{32} \times \frac{51}{1} =$ 10£. 18s. $6\frac{6}{7}$d. Ans.

36. BY ANALYSIS.

If 5yd. cost \$7, 1yd. cost \$$\frac{7}{5}$; and if 7yd. sold for \$11, 1yd. sold for \$$\frac{11}{7}$; therefore $\frac{11}{7} - \frac{7}{5} =$ \$$\frac{6}{35}$ were gained on each yard. And as \$200 were gained, there must have been $200 \div \frac{6}{35} = 1166\frac{2}{3}$yd.; and $1166\frac{2}{3} \div 129\frac{17}{27} = 9$ bales, Ans.

FORM OF STATEMENT.

5yd. : 1yd. : : \$7 : \$$\frac{7}{5}$; 7yd. : 1yd. : : \$11 : \$$\frac{11}{7}$; $\frac{11}{7} - \frac{7}{5} =$ \$$\frac{6}{35}$;
\$$\frac{6}{35}$: \$200 : : 1yd. : $1166\frac{2}{3}$yd.;
$129\frac{17}{27}$yd. : $1166\frac{2}{3}$yd. : : 1 bale : 9 bales, Ans.

37. 13° 10′ 35″ : 360° : : 24h. : 27da. 7h. 43m. + Ans.

38. BY ANALYSIS.

If 7lb. cost \$$\frac{3}{4}$, 1lb. will cost $\frac{1}{7} \times \frac{3}{4} =$ \$$\frac{3}{28}$; and 12lb. will cost $\frac{12}{1} \times \frac{3}{28} =$ \$$\frac{36}{28} =$ \$$1.28\frac{4}{7}$ Ans.

FORM OF STATEMENT.

7lb. : 12lb. : : \$$\frac{3}{4}$: \$$1.28\frac{4}{7}$ Ans.

39. BY ANALYSIS.

If 7lb. cost $1.75, 1lb. will cost $\frac{175}{7}=$ $0.25; and if 25 cents buy 1lb., $213.50 will buy as many pounds as this sum contains 25 cents; thus, $213.50 $\div$ .25 $=$ 854lb. $=$ 7cwt. 2qr. 14lb. Ans.

FORM OF STATEMENT.

$1.75 : $213.50 : : 7lb. : 854lb. $=$ 7cwt. 2qr. 14lb. Ans.

40. BY ANALYSIS.

If 7oz. of gold is worth 30£., 1oz. is worth $\frac{30}{7}$£.; and 7lb. 11oz. $=$ 95oz. is worth $\frac{95}{1}\times\frac{30}{7}=\frac{2850}{7}$£. $=$ 407£. 2s. $10\frac{2}{7}$d. Ans.

FORM OF STATEMENT.

7oz. : 7lb. 11oz. : : 30£. : 407£. 2s. $10\frac{2}{7}$d. Ans.

41. BY ANALYSIS.

$500 for 6 months, is the same as $3000 for 1 month; and if $3000 give 1 month, $600 will give $3000 $\div$ $600 $=$ 5 months, Ans.

FORM OF STATEMENT.

$600 : $500 : : 6m. : 5m. Ans.

42. BY ANALYSIS.

If $8 give 7 oz., $1 would give 56oz., and $$7\frac{1}{2}$ would give $56\div7\frac{1}{2}=7\frac{7}{15}$oz. Ans.

FORM OF STATEMENT.

$7.50 : $8.00 : : 7oz. : $7\frac{7}{15}$oz. Ans.

43. BY ANALYSIS.

Each suit would contain $3\frac{3}{4}\times1\frac{7}{8}=\frac{225}{32}$ square yards; and to clothe the regiment, it would require $\frac{1000}{1}\times\frac{225}{32}=\frac{225000}{32}=7031\frac{1}{4}$yd. To line this cloth, it would require $7031.25\div1.25=5625$yd. Ans.

FORM OF STATEMENT.

1 man : 1000 men :: $3\frac{3}{4} \times 1\frac{7}{8} = \frac{225}{32}$yd. : $7031\frac{1}{4}$yd.;
$1\frac{1}{4}$yd. : 1yd. :: $7031\frac{1}{4}$yd. : 5625yd. Ans.

44. BY ANALYSIS.

If $9\frac{5}{7}$yd. cost $\$11\frac{3}{5}$, 1yd. will cost $11\frac{3}{5} \div 9\frac{5}{7} = \$1\frac{33}{170}$; $16\frac{16}{203}$E. $= 20\frac{20}{203}$yd.; and if 1yd. cost $\$1\frac{33}{170}$, $20\frac{20}{203}$yd. will cost $20\frac{20}{203} \times 1\frac{33}{170} = \$24$ Ans.

FORM OF STATEMENT.

$9\frac{5}{7}$yd. : $16\frac{16}{203}$EE. $= 20\frac{20}{203}$yd. :: $\$11\frac{3}{5}$: \$24 Ans.

45. BY ANALYSIS.

If for \$17280 there be received \$15120 only, for \$1 there will be received $17280 \div 15120 = \$0.87\frac{1}{2}$. A will, therefore, receive $\$5670 \times .87\frac{1}{2} = \$4961.25$ Ans.

FORM OF STATEMENT.

\$17280 : \$15120 :: \$1 : $\$0.87\frac{1}{2}$;
\$1 : $\$0.87\frac{1}{2}$ :: \$5670 : \$4961.25 Ans.

46. BY ANALYSIS.

49 guineas $= 49 \times 28 = 1372$s.; and, if 57yd. cost 1372s., 1yd. will cost $1372 \div 57 = 24\frac{4}{57}$s.; $\frac{1}{4}$ of which is $6\frac{1}{57}$s. = price of 1qr. An ell English will cost 5 times as much; $6\frac{1}{57} \times 5 = 30\frac{5}{57}$s. = 1£. 10s. $1\frac{1}{19}$d. Ans.

FORM OF STATEMENT.

57yd. : 1EE. :: 49guin. : 1£. 10s. $1\frac{1}{19}$d. Ans.

47. BY ANALYSIS.

If \$1.15 buy 1gal., \$100 will buy $100 \div 1.15 = 86$gal. 3qt. $1\frac{15}{23}$pt. Ans.

FORM OF STATEMENT.

\$1.15 : \$100 :: 1gal. : 86gal. 3qt. $1\frac{15}{23}$pt. Ans

48. **BY ANALYSIS.**

If 9 packages cost \$34560, 1 package will cost 34560 ÷ 9 = \$3840; and if 1 parcel cost \$3840 ÷ 8 = \$480, 1 piece will cost \$480 ÷ 12 = \$40, and 1yd. will cost \$40 ÷ 20 = \$2.00 Ans.

FORM OF STATEMENT.

9pack. : 1pack. : : \$34560 : \$3840;
8par. : 1par. : : \$3840 : \$480; 12pieces : 1 piece : : \$480 : \$40;
20yd. : 1yd. : : \$50 : \$2.00 Ans.

49. **BY ANALYSIS.**

If 75 gallons run into the cistern, and 40 gallons run out per hour, there is left in the cistern 35 gallons; and if 35 gallons give 1 hour, 500 gallons will give 500 ÷ 35 = 14h. 17m. $8\frac{4}{7}$sec. Ans.

FORM OF STATEMENT.

75gal. — 40gal. = 35gal.;
35gal. : 500gal. : : 1h. : 14h. 17m. $8\frac{4}{7}$sec. Ans.

50. **BY ANALYSIS.**

If one pair may be bought for \$0.56, then for \$120.96 may be had 12096 ÷ 56 = 216 pair = 18doz. Ans.

FORM OF STATEMENT.

\$0.56 : \$120.96 : : 1 pair : 216 pair = 18doz. Ans.

51. **BY ANALYSIS.**

If the first pipe will empty the cistern in 20 minutes, in 1 minute $\frac{1}{20}$ of it will be emptied. The second pipe will empty $\frac{1}{40}$ of it in 1 minute. The third pipe will empty $\frac{1}{75}$ of it in a minute. Therefore in 1 minute $\frac{1}{20} + \frac{1}{40} + \frac{1}{75} = \frac{53}{600}$ of the cistern will be emptied. And, if $\frac{53}{600}$ of the cistern be emptied in 1 minute, the whole will be emptied in 600 — 53 = 11h. $19\frac{13}{53}$m. Ans.

FORM OF STATEMENT.

$20\text{m.} : 1\text{m.} :: 1\text{cist.} : \frac{1}{20}$
$40\text{m.} : 1\text{m.} :: 1\text{cist.} : \frac{1}{40}$
$75\text{m.} : 1\text{m.} :: 1\text{cist.} : \frac{1}{75}$

$\frac{53}{600}$

$\frac{53}{600}\text{cist.} : 1\text{cist.} :: 1\text{m.} : 11\text{m.}\ 19\frac{13}{53}\text{sec.}$ Ans.

52. BY ANALYSIS.

If A can mow the field in 5 days, in 1 day he will mow $\frac{1}{5}$ of it. B in 1 day will mow $\frac{1}{6}$ of the field. Therefore A and B will, in one day, mow $\frac{1}{5}+\frac{1}{6}=\frac{11}{30}$ of the field; and it will require $30 \div 11 = 2\frac{8}{11}$ days to mow the whole field, Ans.

FORM OF STATEMENT.

$5\text{da.} : 1\text{da.} :: 1 \text{ field} : \frac{1}{5}$; $6\text{da.} : 1\text{da.} :: 1 \text{ field} : \frac{1}{6}$;
$\frac{1}{5}+\frac{1}{6}=\frac{11}{30}$ field : 1 field :: 1 da. : $2\frac{8}{11}$da. Ans.

53. BY ANALYSIS.

To raise the wall 8 feet, it required the labor of 6 men 12 days; that is, the labor of 72 men 1 day; and to raise it 1 foot, it would take $\frac{1}{8}$ of 72 men $= 72 \div 8 = 9$ men; and to raise it the remaining $32 - 8 = 24$ feet in one day, it must require 24 times 9 men; $24 \times 9 = 216$ men. But as they have 6 days to perform the labor, it will take only $\frac{1}{6}$ of their number, $216 \div 6 = 36$ men. Ans.

FORM OF STATEMENT.

8 feet : 24 feet :: 6 men : 18 men.
Then 6 days : 12 days :: 18 men : 36 men, Ans.

54. BY ANALYSIS.

A can do $\frac{1}{20}$ of the labor in 1 day, and A and C can do $\frac{1}{12}$ of it; therefore C alone can do only $\frac{1}{12}-\frac{1}{20}=\frac{1}{30}$ of it. It will therefore take him 30 days to build the boat alone, Ans.

FORM OF STATEMENT.

$\frac{1}{12} - \frac{1}{20} = \frac{1}{30} : 1 :: 1$ day : 30 days, Ans.

55. BY ANALYSIS.

If 700 men are provided with 184000lb., there are for each man $184000 \div 700 = 262\frac{6}{7}$lb. And if each person consume 5lb. per week, he will have sufficient to last him $262\frac{6}{7} \div 5 = 52\frac{4}{7}$ weeks $=$ 52 weeks 4 days, Ans.

FORM OF STATEMENT.

700 men : 1 man :: 184000lb. : $262\frac{6}{7}$lb.

Then 5lb. : $262\frac{6}{7}$lb. :: 1 week : 52 weeks 4 days, Ans.

56. BY ANALYSIS.

Each man has $3 \times \frac{3}{4} = \frac{9}{4}$lb. of beef per week, and 25 men have $25 \times \frac{9}{4} = 56\frac{1}{4}$lb. If, then, $56\frac{1}{4}$lb. last 1 week, 3150lb. will last $3150 \div 56\frac{1}{4} = 56$ weeks, Ans.

FORM OF STATEMENT.

$\frac{3}{4} \times 3 \times 25 = 56\frac{1}{4}$lb. : 3150lb. :: 1 week : 56 weeks, Ans.

57. BY ANALYSIS.

The floor contains $20 \times 16 = 320$ square feet; and $320 \times 144 = 46080$ square inches. The tiles contain $8 \times 8 = 64$ square inches each; and it will therefore require $46080 \div 64 = 720$ tiles, Ans.

FORM OF STATEMENT.

$8 \times 8 = 64$in. : $20 \times 16 \times 144 = 46080$in. :: 1 tile : 720 tiles, Ans.

58. FORM OF STATEMENT.

$10 \times 9 \times 4 = 360$ cubic inches in each stone.

$80 \times 20 \times 2\frac{1}{4} \times 1728 = 6220800$ cubic inches in the wall.

360in. : 6220800in. :: 1 stone : 17280 stones, Ans.

59.

FORM OF STATEMENT.

$3.00 × 60 = $180, price given for the Holland.
$4.00 × 60 = $240, price obtained for it.
$180 : $240 :: $240 : $320 Ans.

60.

BY ANALYSIS.

27 × 7 = 189 = miles A is ahead of B.

36 — 27 = 9 miles that B gains each day on A. If, therefore, 9 miles are gained in one day, it will require, to gain 189 miles, 189 ÷ 9 = 21 days, Ans.

FORM OF STATEMENT.

36 — 27 = 9m. : 189m. :: 1 day : 21 days, Ans.

61.

BY ANALYSIS.

2s. 3d. = 27d., price obtained for the coffee. It is evident that 27d. is $\frac{135}{100}$ of the cost; therefore $\frac{100}{135}$ × 27d. = 20d. was the cost, Ans.

FORM OF STATEMENT.

135d. : 100d. :: 27d. : 20d. Ans.

62.

FORM OF STATEMENT.

2000 × 12 × 7 × 14 = 2352000 ÷ 16 = 147000lb. whole quantity.
105 × 200 = 21000lb. wholly spoiled.
147000 — 21000 = 126000lb. left to subsist on.
2000 × 12 × 7 = 168000 rations.
2016000 ÷ 168000 = 12oz. for each man per day, Ans.

63.

FORM OF STATEMENT.

2000 × 12 × 7 × 12 = 2016000 ÷ 16 = 126000lb. quantity subsisted on.
126000 ÷ 6 = 21000lb. spoiled.
21000 × 7 = 147000lb. the whole quantity, Ans.

64. FORM OF STATEMENT.

$2000 \times 12 \times 7 \times 14 = 2352000 \div 16 = 147000$lb. whole weight.

$2000 \times 12 \times 7 \times 12 = 2016000 \div 16 = 126000$lb. left to subsist on, Ans.

65. FORM OF STATEMENT.

$2000 \times 12 \times 7 \times 12 = 2016000 \div 16 = 126000$lb.

$126000 \div 6 = 21000 =$ quantity lost.

$21000 \times 7 = 147000$lb. whole quantity.

$147000 \times 16 = 2352000 \div \overline{2000 \times 12 \times 7} = 14$oz. Ans.

66. FORM OF STATEMENT.

.85gal. : .25gal. : : $2.72 : $0.80 Ans.

67. FORM OF STATEMENT.

61.3lb. : 1lb. : : $44.9942 : $0.73.4 Ans.

68. FORM OF STATEMENT.

.15hhd. : 1hhd. : : $2.39 : $0.35.85 Ans.

69. FORM OF STATEMENT.

.75 ton : 1 ton : : $15 : $20.00 Ans.

70. FORM OF STATEMENT.

.5yd. : 6yd. : : 10yd. : 120yd. Ans.

71. FORM OF STATEMENT.

10h. : 12h. : : 15 days : 18 days, Ans.

72. FORM OF STATEMENT.

9m. : 5m. : : 450 men : 250 men ; 450—250=200 men, Ans.

73. BY ANALYSIS.

As the hour hand and minute hand pass each other 11 times in 12 hours, and as they are together at 12 o'clock, it is evident that they will next pass each other in $\frac{1}{11}$ of 12 hours $=$ 1h. 5m. $27\frac{3}{11}$sec. Ans.

FORM OF STATEMENT.

11h. : 12h. : : 1h. : 1h. 5m. $27\frac{3}{11}$sec. Ans.

74. BY ANALYSIS.

If A and B can perform a piece of labor in $5\frac{5}{11}$ days, it is evident, that in 1 day they would do $\frac{1}{5\frac{5}{11}} = \frac{11}{60}$ of the work. If B and C can do the work in $6\frac{2}{3}$ days, in 1 day they would perform $\frac{1}{6\frac{1}{2}} = \frac{3}{20}$ of the work. If A and C can do the work in 6 days, in 1 day they would perform $\frac{1}{6}$ of it. It then appears, that A, B, and C, by laboring each 2 days, will perform $\frac{11}{60} + \frac{3}{20} + \frac{1}{6} = \frac{30}{60} = \frac{1}{2}$ of the work; and, therefore, by laboring 1 day each, they would do $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$ of it. And if $\frac{1}{4}$ of the labor be performed in 1 day, it is evident that the whole work will be performed in 4 days, Ans.

If A, B, and C, can do $\frac{1}{4}$ of the work in one day, and A and B can do $\frac{11}{60}$, it is evident that C can do $\frac{1}{4} - \frac{11}{60} = \frac{4}{60} = \frac{1}{15}$ of it in 1 day, and therefore will be 15 days in performing the whole. B and C can do $\frac{3}{20}$ of it in a day; therefore A can do $\frac{1}{4} - \frac{3}{20} = \frac{1}{10}$ of it in a day, or he will be 10 days in doing the whole. A and C can do $\frac{1}{6}$ of the work in a day; therefore B can do $\frac{1}{4} - \frac{1}{6} = \frac{1}{12}$ in a day, or, in performing the whole labor, he will be 12 days, Ans.

FORM OF STATEMENT.

$5\frac{5}{11}$ days : 1 day : : 1 work : $\frac{11}{60}$ work $=$ A and B.
$6\frac{2}{3}$ days : 1 day : : 1 work : $\frac{3}{20}$ work $=$ B and C.
6 days : 1 day : : 1 work : $\frac{1}{6}$ work $=$ A and C.

$\frac{1}{2}$ work $=$ 2 A, 2 B, 2 C $=$
$\frac{1}{4}$ work $=$ A, B, and C.

$\frac{1}{4}-\frac{11}{60}=\frac{1}{15}$ work : 1 work :: 1 day : 15 days = C.
$\frac{1}{4}-\frac{3}{20}=\frac{1}{10}$ work : 1 work :: 1 day : 10 days = A.
$\frac{1}{4}-\frac{1}{6}=\frac{1}{12}$ work : 1 work :: 1 day : 12 days = B.
$\frac{1}{4}$ work : 1 work :: 1 day : 4 days = A, B, and C.

SECTION XLV.

COMPOUND PROPORTION.

(p. 181.)

3.

$$\left.\begin{array}{l}\$100 : \$500\\ 12\text{mo.} : 4\text{mo.}\end{array}\right\} :: \$6 : \$10 \text{ Ans.}$$

4.

$$\left.\begin{array}{l}\$6 : \$10\\ 4\text{mo.} : 12\text{mo.}\end{array}\right\} :: \$100 : \$500 \text{ Ans.}$$

5.

$$\left.\begin{array}{l}\$500 : \$100\\ \$6 : \$10\end{array}\right\} :: 12\text{mo.} : 4\text{mo.} \text{ Ans.}$$

6.

$$\left.\begin{array}{l}\$500 : \$100\\ 4\text{mo.} : 12\text{mo.}\end{array}\right\} :: \$10 : \$6 \text{ Ans.}$$

7.

$$\left.\begin{array}{l}8 \text{ men} : 24 \text{ men}\\ 13 \text{ weeks} : 52 \text{ weeks}\end{array}\right\} :: \$32 : \$384 \text{ Ans.}$$

6

8.

$$\left.\begin{array}{rcr} 60 \text{ men} & : & 12 \text{ men} \\ 30 \text{ feet} & : & 300 \text{ feet} \\ 6 \text{ feet} & : & 8 \text{ feet} \\ 3 \text{ feet} & : & 6 \text{ feet} \\ 8 \text{ hours} & : & 12 \text{ hours} \end{array}\right\} :: 15 \text{ days} : 120 \text{ days, Ans.}$$

9.

$$\left.\begin{array}{rcr} 16 \text{ horses} & : & 32 \text{ horses} \\ 24 \text{ days} & : & 48 \text{ days} \end{array}\right\} :: 84 \text{ bushels} : 336 \text{ bushels, Ans.}$$

10.

$$\left.\begin{array}{rcr} 644\text{lb.} & : & 865\text{lb.} \\ 150 \text{ miles} & : & 64 \text{ miles} \end{array}\right\} :: \$24.58 : \$14.08.6 + \text{ Ans.}$$

11.

7oz. 5dwt. = 145dwt.; 4s. 2d. = 50d.; 5s. 6d. = 66d.;
1s. 2d. = 14d.

$$\left.\begin{array}{rcr} 66\text{d.} & : & 50\text{d.} \\ 4\frac{3}{4}\text{d.} & : & 14\text{d.} \end{array}\right\} :: 145\text{dwt.} : 1\text{lb. } 4\text{oz. } 3\tfrac{479}{627}\text{dwt. Ans.}$$

SECTION XLVI.

CHAIN RULE.

(p. 183.)

2. FORM OF STATEMENT.

12lb. Boston, = 10lb. Amsterdam;
10lb. Amsterdam, = 12lb. Paris;
80lb. Paris.

OPERATION.

$$\frac{12 \times 10 \times 80 = 9600}{10 \times 12 = 120} = 80\text{lb. Ans.}$$

3. FORM OF STATEMENT.

25lb. Boston = 22lb. Nuremburg;
88lb. Nuremburg = 92lb. Hamburg;
46lb. Hamburg = 49lb. Lyons;
98lb. Lyons.

OPERATION.

$$\frac{25 \times 88 \times 46 \times 98 = 9917600}{22 \times 92 \times 49 = 99176} = 100\text{lb. Ans.}$$

4. FORM OF STATEMENT.

24s. Massachusetts = 32s. New York;
48s. New York = 45s. Pennsylvania;
15s. Pennsylvania = 10s. Canada;
100s. Massachusetts.

OPERATION.

$$\frac{32 \times 45 \times 10 \times 100 = 1440000}{24 \times 48 \times 15 = 1728} = 83\tfrac{1}{3}\text{s. Ans.}$$

5. FORM OF STATEMENT.

17 men = 25 women;
5 women = 7 boys;
75 boys.

OPERATION.

$$\frac{17 \times 5 \times 75 = 6375}{25 \times 7 = 175} = 36\tfrac{3}{7}\text{ men, Ans.}$$

6. FORM OF STATEMENT.

10bbls. cider = 5 cords wood;
20 cords wood = 4 tons hay;
50 tons hay.

OPERATION.

$$\begin{array}{r} 10\times 20\times 50= \\ 5\times 4= \end{array}\begin{array}{c} 10000 \\ \hline 20 \end{array}=500\text{bbls. Ans.}$$

7. FORM OF STATEMENT.

100A. Bradford = 120A. Haverhill;
50A. Haverhill = 65A. Methuen;
150A. Methuen.

OPERATION.

$$\begin{array}{r} 100\times 50\times 150= \\ 120\times 65= \end{array}\begin{array}{c} 750000 \\ \hline 7800 \end{array}=96\tfrac{2}{13}\text{A. Ans.}$$

8. FORM OF STATEMENT.

10lb. cheese = 7lb. butter;
11lb. butter = 2bu. corn;
11bu. corn = 8bu. rye;
4bu. rye = 1 cord wood;
10 cords wood.

OPERATION.

$$\begin{array}{r} 10\times 11\times 11\times 4\times 10= \\ 7\times 2\times 8\times 1= \end{array}\begin{array}{c} 48400 \\ \hline 112 \end{array}=432\tfrac{1}{7}\text{lb. Ans.}$$

SECTION XLVII.

COMPANY BUSINESS, OR FELLOWSHIP.

(p. 185.)

2 BY ANALYSIS.

The capital is \$1100. A's stock is \$250; his share of the loss will therefore be $\frac{250}{1100}=\frac{5}{22}$. B's stock is \$300;

his share of the loss therefore is $\frac{300}{1100}=\frac{3}{11}$. C's stock is $550, and his share of the loss therefore is $\frac{550}{1100}=\frac{1}{2}$. The loss, being 5 per cent. of the stock, is \$1100 × .05 = \$55.00. A's loss will therefore be $\frac{5}{22}$ of \$55 = \$12.50 ; B's loss $\frac{3}{11}$ of \$55 = \$15.00 · and C's loss $\frac{1}{2}$ of \$55 = \$27.50 Ans.

FORM OF STATEMENT.

\$1100 × .05 = \$55.00 = loss.

$$\left.\begin{array}{l}\$1100 : \$55.00 :: \$250 : \$12.50 \text{ A's loss,}\\ \$1100 : \$55.00 :: \$300 : \$15.00 \text{ B's loss,}\\ \$1100 : \$55.00 :: \$550 : \$27.50 \text{ C's loss,}\end{array}\right\}\text{Ans.}$$

3. BY ANALYSIS.

The capital is \$18780. C's stock being \$6780, his share of the gain will be $\frac{6780}{18780}=\frac{113}{313}$ of \$1000 = \$361.02.2$\frac{114}{313}$. D's stock is \$12000; his share of the gain will therefore be $\frac{12000}{18780}=\frac{200}{313}$ of \$1000 = \$638.97.7$\frac{199}{313}$ Ans.

FORM OF STATEMENT.

$$\left.\begin{array}{l}\$18780 : \$1000 :: \$6780 \;\; : \$361.02.2\tfrac{114}{313} \text{ C's gain,}\\ \$18780 : \$1000 :: \$12000 : \$638.97.7\tfrac{199}{313} \text{ D's gain,}\end{array}\right\}\text{Ans.}$$

4. BY ANALYSIS.

M's stock is \$3000, and his share of the gain will be $\frac{3000}{10000}=\frac{3}{10}$ of \$500 = \$150. P's stock is \$2000; his share of the gain will therefore be $\frac{2000}{10000}=\frac{1}{5}$ of \$500 = \$100. Q's stock is \$5000; his share of the gain, then, will be $\frac{5000}{10000}=\frac{1}{2}$ of \$500 = \$250 Ans.

FORM OF STATEMENT.

$$\left.\begin{array}{l}\$10000 : \$500 :: \$3000 : \$150 \text{ M's gain,}\\ \$10000 : \$500 :: \$2000 : \$100 \text{ P's gain,}\\ \$10000 : \$500 :: \$5000 : \$250 \text{ Q's gain,}\end{array}\right\}\text{Ans.}$$

5. BY ANALYSIS.

The whole gain is $332.50; but C's gain is $120; A and B's gain, therefore, is $332.50 — $120 = $212.50. A's stock being $500, his share of the gain will be $\frac{500}{850} = \frac{10}{17}$ of $212.50 = $125. B's stock being $350, his share of the gain will be $\frac{350}{850} = \frac{7}{17}$ of $212.50 = $87.50. As the stock of each person in the firm bears the same proportion to his gain as the other; and as A's gain is $125, and his stock $500; therefore,

$125 A's gain : $500 A's stock :: $120 C's gain : $480 C's stock.

Then $480 ÷ 320 = $1.50, value of C's cloth per yard, Ans.

FORM OF STATEMENT.

$850 : $212.50 :: $500 : $125 A's gain, } Ans.
$850 : $212.50 :: $350 : $87.50 B's gain, }
$125 : $500 :: $120 : $480 value of C's stock.
$480 ÷ 320 = $1.50 C's cloth per yard, Ans.

6. BY ANALYSIS.

$5000 + $6500 + $7500 = $19000 amount of stock.
$19000 × .40 = $7600 gross gain.
$7600 × .90 = $6840 net gain.
$\frac{5000}{19000} = \frac{5}{19}$ of $6840 = $1800 A's gain, }
$\frac{6500}{19000} = \frac{13}{38}$ of $6840 = $2340 B's gain, } Ans.
$\frac{7500}{19000} = \frac{15}{38}$ of $6840 = $2700 C's gain, }

FORM OF STATEMENT.

$19000 : $6840 :: $5000 : $1800 A's gain, }
$19000 : $6840 :: $6500 : $2340 B's gain, } Ans.
$19000 : $6840 :: $7500 : $2700 C's gain, }

7. BY ANALYSIS.

Amount of debts, $600 + $760 + $840 + $800 = $3000. For every dollar, then, each creditor will receive $\frac{2275}{3000} =$ $$\frac{91}{120}$.

$$\left.\begin{array}{l}\text{A will receive } \frac{91}{120} \text{ of } \$600 = \$455.00,\\ \text{B will receive } \frac{91}{120} \text{ of } \$760 = \$576.33\frac{1}{3},\\ \text{C will receive } \frac{91}{120} \text{ of } \$840 = \$637.00,\\ \text{D will receive } \frac{91}{120} \text{ of } \$800 = \$606.66\frac{2}{3},\end{array}\right\}\text{Ans.}$$

FORM OF STATEMENT.

$$\left.\begin{array}{l}\$3000 : \$2275 :: \$600 \;.\; \$455.00 \text{ A receives,}\\ \$3000 : \$2275 :: \$760 : \$576.33\frac{1}{3} \text{ B receives,}\\ \$3000 : \$2275 :: \$840 : \$637.00 \text{ C receives,}\\ \$3000 : \$2275 :: \$800 : \$606.66\frac{2}{3} \text{ D receives,}\end{array}\right\}\text{Ans.}$$

8. BY ANALYSIS.

As the bankrupt owes \$5000, and his effects are only \$4000, he will pay on each dollar $\frac{4000}{5000} = \frac{4}{5} = \$0.80$ Ans.

FORM OF STATEMENT.

\$5000 : \$4000 :: \$1.00 : \$0.80 Ans.

9. BY ANALYSIS.

As \$1728 is $\frac{15}{100}$ of the sum owed, that sum must be $\frac{100}{15}$ of $\$1728 = \$11520$ Ans.

FORM OF STATEMENT.

\$0.15 : \$100 :: \$1728 : \$11520 Ans.

SECTION XLVIII.

PARTNERSHIP ON TIME,

OR

DOUBLE FELLOWSHIP.

(p. 187.)

2. BY ANALYSIS.

$3200 for 12 months, is the same as $38400 for 1 month; and $4200 for 8 months, is the same as $33600 for 1 month. Therefore the whole stock may be considered $\$38400+\$33600=\$72000$. A's share of the profit will therefore be $\frac{38400}{72000}=\frac{8}{15}$ of $\$240=\$128$; B's share will be $\frac{33600}{72000}=\frac{7}{15}$ of $\$240=\$112$ Ans.

FORM OF STATEMENT.

$$\begin{array}{l} \$3200\times12=\$38400 \text{ A's product.} \\ \$4200\times\ \,8=\underline{\$33600} \text{ B's product.} \\ \qquad\qquad\quad \$72000 \end{array}$$

$$\left.\begin{array}{l} \$72000:\$240::\$38400:\$128 \text{ A's gain,} \\ \$72000:\$240::\$33600:\$112 \text{ B's gain,} \end{array}\right\}\text{Ans.}$$

BY ANALYSIS.

3. $300 for 5 months, is the same as $1500 for 1 month; $400 for 8 months, is the same as $3200 for 1 month; $500 for 3 months, is the same as $1500 for 1 month. The capital, then, may be considered $\$1500+\$3200+\$1500=\$6200$. A's share of the gain will therefore be $\frac{1500}{6200}=\frac{15}{62}$ of $\$100=\$24.19\frac{11}{31}$; B's share, $\frac{3200}{6200}=\frac{16}{31}$ of $\$100=\$51.61\frac{9}{31}$; C's share, $\frac{1500}{6200}=\frac{15}{62}$ of $\$100=\$24.19\frac{11}{31}$ Ans.

FORM OF STATEMENT.

$$\begin{aligned}\$300 \times 5 &= \$1500\\ \$400 \times 8 &= \$3200\\ \$500 \times 3 &= \$1500\\ \hline &\ \$6200\end{aligned}$$

$$\left.\begin{array}{l}\$6200 : \$100 :: \$1500 : \$24.19\tfrac{11}{31} \text{ A's gain,}\\ \$6200 : \$100 :: \$3200 : \$51.61\tfrac{9}{31} \text{ B's gain,}\\ \$6200 : \$100 :: \$1500 : \$24.19\tfrac{11}{31} \text{ C's gain,}\end{array}\right\}\text{Ans.}$$

4. 24 oxen for 8 weeks, is 1 ox for 192 weeks; 18 oxen for 12 weeks, is 216 oxen for 1 week; and 12 oxen for 10 weeks, is 1 ox for 120 weeks. The amount of pasturing is $192+216+120=528$ weeks. A's share is $\frac{192}{528}=\frac{4}{11}$ of $\$26.40=\$9.60$; B's share is $\frac{216}{528}=\frac{9}{22}$ of $\$26.40=\$10.80$; C's share is $\frac{120}{528}=\frac{5}{22}$ of $\$26.40=\$6.00$ Ans.

FORM OF STATEMENT.

$$\begin{aligned}24 \text{ oxen} \times 8 &= 192 \text{ oxen};\\ 18 \text{ oxen} \times 12 &= 216 \text{ oxen};\\ 12 \text{ oxen} \times 10 &= 120 \text{ oxen};\\ \hline &\ 528 \text{ oxen.}\end{aligned}$$

$$\left.\begin{array}{l}528 : \$26.40 :: 192 : \ \$9.60 \text{ what A pays,}\\ 528 : \$26.40 :: 216 : \$10.80 \text{ what B pays,}\\ 528 : \$26.40 :: 120 : \ \$6.00 \text{ what C pays,}\end{array}\right\}\text{Ans.}$$

5. The "stock in trade" is a carriage to ride 144 miles; and the expense for the carriage, \$25, may be considered the "loss," and the proportional part which each rode, the *time*. Now, by the Rule, each man is to bear his share of the loss (expense) in proportion as he has the use of the stock in trade (carriage). The two men had the use of the whole stock in trade for the first 20 miles, for which they pay $\frac{20}{144}$ of $\$25=\$3.47.2\frac{2}{9}$; therefore, the share of each, for this distance, is $\frac{1}{2}$ of $\$3.47.2\frac{2}{9}=\$1.73.6\frac{1}{9}$. For the next 52

miles the carriage was occupied by the two gentlemen and A. The expense of the carriage for this distance was \$9.02.7$\frac{7}{9}$, it being $\frac{52}{144}$ of \$25, of which each paid $\frac{1}{3}$ = \$3.00.9$\frac{7}{27}$. For the next 42 miles, there were four passengers, viz. the two gentlemen, and A and B. The expense for this distance was $\frac{42}{144}$ of \$25 = \$7.29.1$\frac{2}{3}$, and each of the four passengers paid \$1.82.2$\frac{11}{12}$. For the remaining part of the distance, 30 miles, the expense was $\frac{30}{144}$ of \$25 = \$5.20.8$\frac{1}{3}$; and as C has been received into the carriage, each person bears $\frac{1}{5}$ of the expense for this distance, \$1.04.1$\frac{2}{3}$. Thus we perceive that each of the two gentlemen pays \$1.73.6$\frac{1}{9}$ + \$3.00.9$\frac{7}{27}$ + \$1.82.2$\frac{11}{12}$ + \$1.04.1$\frac{2}{3}$ = \$7.60.9$\frac{103}{108}$; A pays \$3.00.9$\frac{7}{27}$ + \$1.82.2$\frac{11}{12}$ + \$1.04.1$\frac{2}{3}$ = \$5.87.3$\frac{91}{108}$; B pays \$1.82.2$\frac{11}{12}$ + \$1.04.1$\frac{2}{3}$ = \$2.86.4$\frac{7}{12}$; C pays \$1.04.1$\frac{2}{3}$ Ans.

FORM OF STATEMENT.

$\overline{\frac{20}{144}\times\frac{1}{2}}+\overline{\frac{52}{144}\times\frac{1}{3}}+\overline{\frac{42}{144}\times\frac{1}{4}}+\overline{\frac{30}{144}\times\frac{1}{5}}=\frac{526}{1728}$, first man's product.

$\overline{\frac{20}{144}\times\frac{1}{2}}+\overline{\frac{52}{144}\times\frac{1}{3}}+\overline{\frac{42}{144}\times\frac{1}{4}}+\overline{\frac{30}{144}\times\frac{1}{5}}=\frac{526}{1728}$, 2d man's product.

$\overline{\frac{52}{144}\times\frac{1}{3}}+\overline{\frac{42}{144}\times\frac{1}{4}}+\overline{\frac{30}{144}\times\frac{1}{5}}=\frac{406}{1728}$, A's prod.

$\overline{\frac{42}{144}\times\frac{1}{4}}+\overline{\frac{30}{144}\times\frac{1}{5}}=\frac{198}{1728}$, B's prod.

$\frac{30}{144}\times\frac{1}{5}=\frac{72}{1728}$, C's prod.

$\overline{\frac{1728}{1728}}$, sum of the products.

$$\left.\begin{array}{l}
\frac{1728}{1728} : \$25 :: \frac{526}{1728} : \$7.60.9\frac{103}{108}, \text{ first man's expense,}\\
\frac{1728}{1728} : \$25 :: \frac{526}{1728} : \$7.60.9\frac{103}{108}, \text{ 2d man's expense,}\\
\frac{1728}{1728} : \$25 :: \frac{406}{1728} : \$5.87.3\frac{91}{108}, \text{ A's expense,}\\
\frac{1728}{1728} : \$25 :: \frac{198}{1728} : \$2.86.4\frac{7}{12}, \text{ B's expense,}\\
\frac{1728}{1728} : \$25 :: \frac{72}{1728} : \$1.04.1\frac{2}{3}, \text{ C's expense,}
\end{array}\right\} \text{Ans.}$$

6. \$4000 for 4 months, is \$100 for 160 months. \$4000 + \$500 = \$4500 for 12 months, is \$100 for 540 months; and \$4500 — \$1000 = \$3500 for 4 months, is \$100 for 140 months. A may therefore be considered as

having $100 in trade 160 + 540 + 140 = 840 months. Again, $3000 for 10 months, is $100 for 300 months. $3000 — $1500 = $1500 for 4 months, is $100 for 60 months. $1500 + $3000 = $4500 for 6 months, is $100 for 270 months. B may therefore be considered as having $100 in trade for 300 + 60 + 270 = 630 months. Again, $2000 for 6 months, is $100 for 120 months. $2000 + $2000 = $4000 for 8 months, is $100 for 320 months. $4000 + $2000 = $6000 for 2 months, is $100 for 120 months. $6000 — $1500 = $4500 for 4 months, is $100 for 180 months. C may therefore be considered as having $100 in trade 120 + 320 + 120 + 180 = 740 months. The sum of A, B, and C's time is 840 + 630 + 740 = 2210 months. A's share of the gain will therefore be $\frac{840}{2210}$ of $4420 = $1680; B's share, $\frac{630}{2210}$ of $4420 = $1260; C's share $\frac{740}{2210}$ of $4420 = $1480 Ans.

FORM OF STATEMENT.

$4000 × 4 = 16000
500
4500 × 12 = 54000
1000
3500 × 4 = 14000
A's product = 84000

$3000 × 10 = 30000
1500
1500 × 4 = 6000
3000
4500 × 6 = 27000
B's product = 63000

$2000 × 6 = 12000
2000
4000 × 8 = 32000
2000
6000 × 2 = 12000
1500
4500 × 4 = 18000
C's product = 74000

A's product,	84000
B's product,	63000
C's product,	74000
	221000

$221000 : $4420 :: $84000 : $1680 A's gain,
$221000 : $4420 :: $63000 : $1260 B's gain, } Ans.
$221000 : $4420 :: $74000 : $1480 C's gain,

7. $4000 for 6 months, is $100 for 240 months. $4000 — $2000 = $2000 for 18 months, is $100 for 360 months. Jones may therefore be considered as having $100 in trade for 240 + 360 = 600 months. $3500 for 8 months, is $100 for 280 months. $3500 — 1500 = $2000 for 16 months, is $100 for 320 months. Eaton, then, may be considered as having $100 in trade 280 + 320 = 600 months. $2500 for 10 months, is $100 for 250 months. $2500 + $2000 = $4500 for 14 months, is $100 for 630 months. Brown, then, may be considered as having $100 in trade 250 + 630 = 880 months. As each man is supposed to have the same capital, his share of the gain will be in proportion to the *time* his $100 has been in trade.

The sum of all their times is 600 + 600 + 880 = 2080 months.

Jones's gain will be $\frac{600}{2080}$ of $1041.80 = 300.51\frac{12}{13}$
Eaton's gain will be $\frac{600}{2080}$ of $1041.80 = 300.51\frac{12}{13}$ } Ans
Brown's gain will be $\frac{880}{2080}$ of $1041.80 = 440.76\frac{2}{13}$

FORM OF STATEMENT.

$4000 × 6 = 24000
2000
2000 × 18 = 36000
Jones's product = 60000

$3500 × 8 = 28000
1500
2000 × 16 = 32000
Eaton's product = 60000

$2500 × 10 = 25000
2000
4500 × 14 = 63000
Brown's product = 88000

60000
60000
88000
Sum of the products = 208000

$208000 : \$1041.80 :: 60000 : \$300.51\frac{12}{13}$ Jones's gain,
$208000 : \$1041.80 :: 60000 : \$300.51\frac{12}{13}$ Eaton's gain,
$208000 : \$1041.80 :: 88000 : \$440.76\frac{2}{13}$ Brown's gain,
Ans.

SECTION L.

LOSS AND GAIN.

2. (p. 190.) $\$7.00 : \$6.12.5 :: \$100 : \$87.50$; $\$100 - \$87.50 = \$12.50$ per cent. Ans.

3. $\$1.00 - .12.5 = .87.5$; $\$7.00 \times .87.5 = \$6.12\frac{1}{2}$ Ans.

4. $\$3.60 \times \$1.12\frac{1}{2} = \$4.05$ Ans.

5. \$4.05 is $\frac{100}{112\frac{1}{2}}$ of the original cost; therefore, $\frac{100}{112\frac{1}{2}}$ of $4.05 = \$3.60$ Ans.

6. $\$8.50 \times 1.10 = \$9.35$ Ans.

7. $\$9.35 \div 1.10 = \$8.50$ Ans.

8. $\$1.00 - .15 = .85$; $\$1.25 \div .85 = \$1.47\frac{1}{17}$; $\$1.47\frac{1}{17} \times 1.12 = \$1.64.7\frac{1}{17}$ Ans.

9. $\$1.00 - .15 = .85$; $\$1.25 \div .85 = \$1.47\frac{1}{17}$, value of the wine; $\$1.64.7\frac{1}{17} - \$1.47\frac{1}{17} = \$0.17.6\frac{8}{17}$; $\$1.47\frac{1}{17} : \$0.17.6\frac{8}{17} :: \$100 : \$12$ per cent. Ans.

10. As 12 per cent. was gained on the wine, $\frac{112}{100}$ of $\$1.64.7\frac{1}{17} = \$1.47\frac{1}{17}$ = cost; and $\frac{85}{100} \times \$1.47\frac{1}{17} = \$1.25$ = price for which it is sold. Ans.

11. $\$1.64.7\frac{1}{17} \div 1.12 = \$1.47\frac{1}{17} - \$1.25 = \$0.22\frac{1}{17}$; $\$1.47\frac{1}{17} : \$0.22\frac{1}{17} :: \$100 : \$15$ per cent. Ans.

12. $\$0.90 : \$1.20 :: \$100 : \$133\frac{1}{3} - \$100 = \$33\frac{1}{3}$ per cent.; $\$1.12\frac{1}{2} : \$1.50 :: \$100 : \$133\frac{1}{3} - \$100 = \$33\frac{1}{3}$ per cent. Both have gained $\$33\frac{1}{3}$ per cent. Ans.

13. The present worth of 13 cents, due 8 months hence, is $.13 \div 1.04 = .12\frac{1}{2}$ cents. If, therefore, the cotton cost

$12\frac{1}{2}$ cents, and is sold for 12 cents, on each yard there is lost $.125 - .12 = .005 = \frac{5}{125} = .04 = 4$ per cent. Ans.

14. $\$2.50 \times 24 = \$60.00$, price sold at. As the cloth was sold at $7\frac{1}{2}$ per cent. loss, it is evident \$60.00 is $\frac{92\frac{1}{2}}{100}$ of the cost; therefore, $\frac{100}{92\frac{1}{2}}$ of $\$60.00 = \$64.86.4\frac{32}{37}$ is the cost, Ans.

15. It is evident, if $7\frac{1}{2}$ per cent. be taken from any sum, $92\frac{1}{2}$ per cent. will remain; therefore, $.92\frac{1}{2} \times \$64.86.4\frac{32}{37} = \$60$, price sold at; $\$60.00 \div 24 = 2.50$ per yard, Ans.

16. By the last question, we perceive, that, if $7\frac{1}{2}$ per cent. be lost on $\$64.86.4\frac{32}{37}$, \$60 will be left; and if this be divided by \$2.50, we shall have the number of yards; thus, $\$60 \div \$2.50 = 24$ yards, Ans.

17. $\$64.86.4\frac{32}{37}$ was given for the cloth, and $\$2.50 \times 24 = \$60.00$ was received for it; therefore the loss was $\$64.86.4\frac{32}{37} - \$60.00 = \$4.86.4\frac{32}{37}$, or $\frac{4864\frac{32}{37}}{60000}$ of its value, $= .075 = 7\frac{1}{2}$ per cent. Ans.

18. If 17 per cent. be added to any sum, it is evident the original sum must be $\frac{100}{117}$ of the amount; therefore, $\frac{100}{117}$ of $\$12.50 = \$10.68.3\frac{89}{117}$ = the first cost, Ans.

19. If, on any sum, 25 per cent. has been lost, the original sum must have been $\frac{100}{75}$ of it. Therefore, $\frac{100}{75}$ of $\$75 = \$100$ was the price of the horse. But his real value was $\$100 \times 1.30 = \$130$; therefore he was sold for $\$130 - \$75 = \$55$ less than his value, Ans.

20. As the horse was worth 30 per cent. more than was given for him, and as he was sold for 25 per cent. less than he cost, these relative prices may be expressed thus; $\frac{130}{100}$, $\frac{75}{100}$; and their difference thus; $\frac{130}{100} - \frac{75}{100} = \frac{55}{100}$. Therefore, $\frac{55}{100} : \frac{75}{100} :: \$55 : \$75$ Ans.

21. $\$0.42 \times .95 = \$0.39.9$ Ans.

22. 63gal. — 15gal. = 48gal. remaining; $\$112 \div 48 = \$2.33\frac{1}{3} \times .95 = \$2.21.6\frac{2}{3}$ Ans.

23. \$112 × .95 = \$106.40 received for the wine; $\$106.40 \div \$2.21.6\frac{2}{3} = 48$gal. remaining; 63gal.—48gal.= 15gal. leaked out, Ans.

24. 63gal. — 15gal. = 48gal. left; $\$2.21.6\frac{2}{3} \times 48 =$ \$106.40, $\$106.40 \div .95 = \$112$ Ans.

25. 63gal. — 15gal. = 48gal.; $\$2.21.6\frac{2}{3} \times 48 = \$106.40$ price sold at; \$112 — \$106.40 = \$5.60, loss;

\$112 : \$5.60 :: \$100 : \$5, loss per cent. Ans.

26. $\$5.60 \div .93 = \$6.02\frac{14}{93}$ value per yard; \$6.25 — $\$6.02\frac{14}{93} = \$0.22\frac{79}{93}$ gain on each yard; and $\frac{22\frac{79}{93}}{602\frac{14}{93}} = \frac{17}{448} = .03\frac{89}{93} = \$3\frac{89}{112}$ gain per cent. Ans.

Or, $\$6.02\frac{14}{93} : \$0.22\frac{79}{93} :: \$100 : \$3\frac{89}{112}$ gain per cent. Ans.

27. $\$35 \div 1.04 = \$33.65.3\frac{11}{13}$ present worth of \$35; $\$33.65.3\frac{11}{13} - \$30 = \$3.65.3\frac{11}{13}$ gain, Ans.

28. $\$1.25 \div .75 = \$1.66\frac{2}{3}$ value of the tea; $\$1.66\frac{2}{3} - \$1.40 = \$0.26\frac{2}{3}$ loss per pound. Therefore, $\frac{26\frac{2}{3}}{166\frac{2}{3}} = \frac{8}{50} =$ .16 = \$16 per cent. loss, Ans.

Or, $\$1.66\frac{2}{3} : \$0.26\frac{2}{3} :: \$100 : \$16$ per cent. loss, Ans.

29. \$1.35 × .88 = \$1.18.8 price for $\frac{1}{3}$ in ready money; $\$1.18.8 : \$1.00 :: \$5.00 : \$4.20\frac{260}{297}$ cash price of 1 yard; \$1.18.8 × 50 = \$59.40 value of the indigo; $\$59.40 \div 3 =$ \$19.80; \$59.40—\$19.80 = \$39.60; $\$39.60 \div \$5.00 = 7\frac{23}{25}$ yards delivered by B. Ans.

SECTION LI.

DUODECIMALS.

9. (p. 193.) 20ft. + 14ft. 6′ = 34ft. 6′ × 2 = 69ft. × 10ft. 4′ = 713ft.; 3ft. 2′ × 6ft. × 2 = 38ft.; 4ft. 4′ × 4ft.=

17ft. 4′ + 38ft. = 55ft. 4′; 713ft. — 55ft. 4′ = 657ft. 8′ ÷ 9 = $73\frac{2}{27}$ yards, Ans.

10. 53ft. 6′ × 10ft. 3′ × 2ft. = 1096ft. 9′ Ans.

11. 6ft. 8′ + 5ft. 9′ + 4ft. 6′ + 3ft. 10′ = 20ft. 9′ × 3ft. 5′ × 4 = 283ft. 7′ Ans.

12. 3ft. 4′ + 2ft. 10′ = 6ft. 2′ × 2 = 12ft. 4′ × 7ft. 9′ = 95ft. 7′; 3ft. 4′ × 2ft. 10′ × 2 = 18ft. 10′ 8″ + 95ft. 7′ = 114ft. 5′ 8″ × 15 = 1717ft. 1′ square feet; 3ft. 4′ × 2ft. 10′ × 7ft. 9′ × 15 = 1097ft. 11′ ÷ 27 = $40\frac{215}{324}$ cubic yards, Ans.

13. 20ft. + 16ft. 6′ = 36ft. 6′ × 2 = 73ft. × 9ft. 6′ = 693ft. 6′; 20ft. × 16ft. 6′ = 330ft.; 330ft. + 693ft. 6′ = 1023ft. 6′ × 3 = 3070ft. 6′ ÷ 9 = 341yd. 1ft. 6′; 341yd. 1ft. 6′ — 90yd. = 251yd. 1ft. 6′ Ans.

14. 17ft. 6′ × 1ft. 7′ = 27ft. 8′ 6″ Ans.

15. 27ft. 9′ × 2ft. 5′ = 67ft. 0′ 9″ Ans.

16. 47ft. × 17ft. 9′ = 834ft. 3′ Ans.

17. 18ft. 9′ × 1ft. 6′ × 3 = 84ft. 4′ 6″ Ans.

18. 20ft. × 1ft. 6′ × $2\frac{1}{2}$ = 75ft. Ans.

19. 40ft. 6′ × 2ft. 6′ × $2\frac{3}{4}$ = 278ft. 5′ 3″ Ans.

20. 18ft. × 6ft. × 4ft. = 432ft. ÷ 128 = $3\frac{3}{8}$ cords, Ans.

21. 10ft. × 5ft. × 7ft. = 350ft. ÷ 128 = 2 cords 94 cubic feet, Ans.

22. 35ft. × 4ft. × 4ft. = 560ft. ÷ 128 = $4\frac{3}{8}$ cords, Ans.

23. 8ft. × 8ft. × 8ft. = 512ft. ÷ 128 = 4 cords, Ans.

24. 10ft. × 10ft. × 10ft. = 1000ft. ÷ 128 = $7\frac{13}{16}$ cords, Ans.

25. 70ft. 6′ × 5ft. 3′ = 370ft. 1′ 4″ ÷ 4 = $92\frac{51}{96}$ cord feet; $92\frac{51}{96}$ ÷ 8 = $11\frac{145}{256}$ cords, Ans.

26. 97ft. 9′ × 3ft. 6′ = 342ft. 1′ 6″ ÷ 4 = $85\frac{51}{256}$ cord feet ÷ 8 = $10\frac{177}{256}$ cords, Ans.

27. 100ft. × 6ft. 11′ = 691ft. 8′ ÷ 4 = $172\frac{11}{12}$ cord feet ÷ 8 = $21\frac{59}{96}$ cords, Ans.

28. 8ft. × 4ft. × 10 = 320ft. × 2′ = $53\frac{1}{3}$ft. × 5 = $266\frac{2}{3}$ ÷ 128 = \$2.08$\frac{1}{3}$ Ans.

29. 3ft. 8′ × 5ft. = 18ft. 4′ ÷ 2 = $9\frac{1}{6}$ feet, Ans.

30. 3ft. 10′ × 6ft. 6′ = 24ft. 11′ ÷ 2 = $12\frac{11}{24}$ feet, Ans.

31. 128ft. ÷ 8 = 16ft. ÷ 3ft. 11′ = 4ft. 6′ $10\frac{2}{7}$″ Ans.

32. 128ft. × 2 = 256ft. ÷ 12 = 21ft. 4′ ÷ 3ft. 9′ = 5ft. 8′ $3\frac{1}{5}$″ Ans.

	£.	s.	d.	qr.
34.	1	19	11	3
	1	19	11	3
	1	19	11	3
	1	17	11	$3\frac{1}{20}$
		1	9	$3\frac{228}{240}$
			1	$1\frac{957}{960}$

Ans. 3£. 19s. 11d. $0\frac{1}{960}$qr.

NOTE.—The first product is obtained by multiplying the multiplicand by 1, the second product by multiplying it by $\frac{18}{20}$, the third product by multiplying by $\frac{11}{240}$, and the fourth product by multiplying by $\frac{3}{960}$.

	m.	fur.	rd.			
35.	3	4	12			
	2	6	8			
	6	8	24			
	2	5	9			
			28	4	11	$1\frac{1}{5}$

Ans. 9m. 6fur. 21rd. 4ft. 11in. $1\frac{1}{5}$br.

NOTE.—The multiplicand is first multiplied by 2, then by $\frac{6}{8}$, and lastly by $\frac{8}{320}$.

36. 71° 4′
4′

Ans. 4h. 44′ 16″

38. 14£. 5s. + 13£. 10s. + 11£. 5s. = 39£. 0s.; and
46£. 5s. — 39£. 0s. = 7£. 5s.

18s. × 3 = 2£. 14s.; 7£. 5s. — 2£. 14s. = 4£. 11s. gain
4£. 11s. ÷ 39 = 2s. 4d. gain on each £.

£.	s.	d.
14	5	0
	2	4
1	8	6
	4	9

Ans. A's gain = 1£. 13s. 3d.

£.	s.	d.
13	10	0
	2	4
1	7	0
	4	6

Ans. B's gain = 1£. 11s. 6d.

£.	s.	d.
11	5	0
	2	4
1	2	6
	3	9

Ans. C's gain = 1£. 6s. 3d.

SECTION LIII.

SQUARE ROOT.

32. (p. 202.) $\sqrt{141376} = 376$ Ans.

33. $1760 \times 9 = 15840$; $\sqrt{15840} = 125.857 +$ feet, Ans.

34. $24 \times 24 = 576 \div 2 = 288$; $\sqrt{288} = 16.97 +$ in. Ans.

35. $2:3::2400:3600$; $\sqrt{3600}=60$ trees in length. $3:2::2400:1600$; $\sqrt{1600}=40$ trees in breadth. $60-1=59\times7=413$; $40-1=39\times7=273$; $413\times273=112749$ square yards, Ans.

36. $2:3::\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}:\frac{27}{32}=.84375$; $\sqrt{.84375}=.918+$ in. Ans.

37. $2\times2=4:1.5\times1.5=2.25::50\text{m.}:28.125\text{m.}=28\text{m.}\ 7\frac{1}{2}\text{sec.}$ Ans.

38. $4\times4\times3=48:6\times6\times2=72::4\text{h.}:6\text{h.}$ Ans.

39. $144\times144=20736$; $64\times64=4096$; $20736-4096=16640$; $\sqrt{16640}=128.99+$ feet, Ans.

40. $20\times20=400$; $16\times16=256$; $12\times12=144$; $400+256+144=800$; $\sqrt{800}=28.28+$ feet, Ans.

41. $128\times128=16384$; $72\times72=5184+16384=21568$; $\sqrt{21568}=146.86+$ miles, Ans.

42. $100\times100=10000$; $70-5=65$; $65\times65=4225$; $10000-4225=5775$; $\sqrt{5775}=75.993420+$; $80\times80=6400$; $50-5=45$; $45\times45=2025$; $6400-2025=4375$; $\sqrt{4375}=66.143782+$; $75.993420+66.143782=142.137202\times142.137202=20202.984192388804$; $70-50=20$; $20\times20=400$; $20202.984192388804+400=20602.984192388804$; $\sqrt{20602.984192388804}=143.537396+$ feet, Ans.

43. $400\times400=160000$; $160\times160=25600$; $160000-25600=134400$; $\sqrt{134400}=366.6-50=316.6$ Ans.

44. $70\times70=4900$; $40\times40=1600$; $30\times30=900$; $4900-1600=3300$; $\sqrt{3300}=57.445+$; $4900-900=4000$; $\sqrt{4000}=63.245+57.445=120.69+$ feet, Ans.

45. $\frac{1}{10}\times\frac{1}{10}=\frac{1}{100}:1^2\text{in.}::450\text{lb.}:45000\text{lb.}$ Ans.

46. $80\times80=6400$; $40\times40=1600$; $80\times2=160$; $6400-1600=4800$; $4800\div160=30$ height of the stump, Ans.

47. Circles being to each other as the squares of their diameters, they are also to each other as the squares of

their semidiameters. The grindstone being 48 inches in diameter, its semidiameter will be 24 inches. $24 \times 24 = 576 \div 4 = 144$; $576 - 144 = 432$, $\sqrt{432} = 20.78 =$ to the semidiameter that will be *left* after A has ground off his share; therefore his share will be $24 - 20.78 = 3.22$in.; $432 - 144 = 288$; $\sqrt{288} = 16.97$; $20.78 - 16.97 = 3.81$in., B's share; $288 - 144 = 144$; $\sqrt{144} = 12$in.; $16.97 - 12 = 4.97$in., C's share; $\sqrt{144} = 12$in., D's share, Ans.

48. $160 - .785398 = 203.71836$; $\sqrt{203.71836} = 14.273 +$; $14.273 \div 2 = 7.136 +$ rods, Ans.

SECTION LIV.

CUBE ROOT.

17. (p. 207.) $3 \times 3 \times 3 = 27 : 6 \times 6 \times 6 = 216 :: 4$lb. : 32lb. Ans.

18. $1 \times 1 \times 1 = 1 : 3.5 \times 3.5 \times 3.5 = 42.875 ::$ \$120 : \$5145 Ans.

19. 5ft. 10in. = 70in.; 10ft. $4\frac{3}{5}$in. = 124.6in.; $70 \times 70 \times 70 = 343000$; $124.6 \times 124.6 \times 124.6 = 1934434.936$; $343000 : 1934434.936 ::$ 180lb. : 1015.1 + lb. Ans.

20. 2lb. : 2000lb. :: $4 \times 4 \times 4 = 64$in. : 64000;
$\sqrt[3]{64000} = 40$in. = 3ft. 4in. high;
2lb. : 2000lb. :: $3 \times 3 \times 3 = 27$in. : 27000in.;
$\sqrt[3]{27000} = 30$in. = 2ft. 6in. wide;
2lb. : 2000lb. :: $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64} : \frac{1000}{64}$;
$\sqrt[3]{\frac{1000}{64}} = \frac{10}{4} = 2\frac{1}{2}$in. thick, Ans.

21. $5 \times 5 \times 5 = 125$ft. : $20 \times 20 \times 20 = 8000$ft. :: 1cwt. : 64cwt. Ans.

22. $6\times6\times6=216$ft. : $10\times10\times10=1000$ft. : : 1 day : 4.629 + days, Ans.

23. $6\times6\times6=216$ft. : $8\times8\times8=512$ft. : : 600lb. : 1422.2 + lb. Ans.

SECTION LV.

ARITHMETICAL PROGRESSION.

2. (p. 210.) $\frac{45-5}{11-1}=4$ miles, Ans.

3. $\frac{48-3}{10-1}=5$ years, Ans.

4. $\frac{39-3}{19-1}=2$ years, Ans.

6. $\frac{\overline{51+7}\times 12}{2}=348$ miles, Ans.

7. $\frac{\overline{39+3}\times 19}{2}=399$ years, Ans.

8. $320\times30=9600=$ rods in 30 miles; but there will be one more stone in this distance than there are rods, because there will be a stone at each end of the 30 miles. The man must travel 2 rods to bring the first stone to the basket, and 60 miles and 2 rods to bring the last stone; wherefore the following formula:

$\frac{\overline{19202+2}\times 9601}{2}=92188802$rd. $=288090$ miles 2 rods, Ans.

10. $\frac{39-3}{2}+1=19$ scholars, Ans.

11. $\frac{51-7}{4}+1=12$ days, Ans.

12. $\frac{\overline{45+3}\times\overline{\overline{45-3}+2}}{2\times2}=528$ sum of the series, Ans.

13. $\frac{\overline{618+6}\times\overline{618-6}+12}{2\times12}=$ \$162.24 Ans.

14. $\frac{45+3\times45-3}{\overline{528\times2}-48}=2$ common difference, Ans.

15. $\frac{528\times2}{22}-3=$ \$45; $\frac{45-3}{22-1}=$ \$2 difference, Ans.

SECTION LVI.

GEOMETRICAL SERIES.

3. (p. 214.) $\frac{72}{3^{6-1}}=\frac{8}{27}$ first term, Ans.

4. $2\times2^{30-1}=$ \$10737418.24 Ans.

5. $5\times3^{7-1}=3645$ seventh term, Ans.

6. $50\times1.06^{5-1}=63.123848$ last term, Ans.

7. \$160 $\times1.06^{7-1}=$ \$226.96.3.05796096 Ans.

8. \$300 $\times1.05^{9-1}=$ \$543.23 6 + Ans.

9. \$100 $\times1.06^{31-1}=$ \$574.34.911729132501162641-0633231080264584635725219606935738777 6 Ans.

11. $\frac{10^{12}-1}{10-1}\times1=$ \$111111111111 Ans.

12. $\frac{2^{40}-1}{2-1}\times10=$ \$10995116277.50 Ans.

13. $\frac{1.06^{4}-1}{1.06-1}\times50=218.7308$ sum of the series, Ans.

14. By examining this question, we find there have been 21 deposites. The amount of the last deposite is \$10.60, the 10 dollars being on interest only one year. The last but one is \$11.23.6. The last but two is \$11.91.0.16. The last but three is \$12.62.4.7696, and so on. Thus we have a

regular geometrical series, the ratio of which is 1.06, the first term \$10.60, the number of terms 21, to find the sum of all the series.

$$\frac{1.06^{21}-1}{1.06-1}\times 10.60=\$423.92.2+ \text{ Ans.}$$

15. $\frac{7-7\times\frac{1}{4}^{5}}{1-\frac{1}{4}}=\frac{7-\frac{7}{1024}}{\frac{3}{4}}=9\frac{83}{256}$ sum of the series, Ans.

16. $2\times 2^{153-1}=$ \$114179815416476790484662877555-95961091061972.99.2 Ans.

Note.—This question should be performed by the rule under Problem I.

SECTION LVII.

INFINITE SERIES

2. (p. 215.) $\frac{5\times 5}{4}=6\frac{1}{4}$ Ans.

3. $\frac{8\times 7}{7-1}=9\frac{1}{3}$ Ans.

4. $\frac{1\times 3}{3-1}=1\frac{1}{2}$ Ans.

5. $\frac{11\times 7}{7-1}=12\frac{5}{6}$ Ans.

6. $\frac{\frac{2}{3}\times 2}{2-1}=1\frac{1}{3}$ Ans.

SECTION LVIII.

DISCOUNT BY COMPOUND INTEREST.

2. (p. 216.) $1.06^4 = 1.26247696$) \$500.00 (\$396.04.6+ Ans.

3. $1.05^{10} = 1.628894$) \$1000.00.0 (\$613.91.3 + Ans.

4. $1.06^2 = 1.1236$) \$800.00.0 (\$711.99.7 +

\$800.00.0 — \$711.99.7 + = \$88.00.3 Ans.

SECTION LIX.

ANNUITIES AT COMPOUND INTEREST.

2. (p. 217.) $\dfrac{1.06^5 - 1}{1.06 - 1} \times \$500 = \$2818.54.6 +$ Ans.

3. $\dfrac{1.06^{10} - 1}{1.06 - 1} \times \$1000 = \$13180.79.4 +$ Ans.

4. $\dfrac{1.06^3 - 1}{1.06 - 1} \times \$30 = \$95.50.8 +$ Ans.

7. $\dfrac{1.06^{10} - 1}{1.06 - 1} \times \$500 = \$6590.39.75$ amount of an annuity of \$500 for 10 years; therefore

$1.06^{10} = 1.7908476$) \$6590.39.75 (\$3680.04 + Ans.

8. $\dfrac{1.05^7 - 1}{1.05 - 1} \times \$200 = \$1628.40.169 +$ amount of an annuity of \$200 for 7 years at 5 per cent. To find the present worth of this sum, we divide it by the amount of \$1 for the given rate and time. Thus the amount of \$1 is $1.05^7=\$1.4071+$; therefore

\$1.4071) \$1628.40.169 (\$1157.27 + Ans.

Or, \$1.4071 : \$1.00 :: \$1628.40.169 : \$1157.27+ Ans.

9. $\frac{1.06^8-1}{1.06-1}\times 300=\$2969.23.9+$ amount of \$300 for 8 years. $1.06^8=\$1.593848+$ amount of \$1 for 8 years. \$1.593848 : \$1.00 :: \$2969.23.9 : \$1862.93.8 Ans.

10. $\frac{1.06^9-1}{1.06-1}\times\$100=\$1149.13.16+$ amount of \$100 for 9 years. $1.06^9=\$1.689478959$ amount of \$1 for 9 years. \$1.689478959 : \$1.00 :: \$1149.13.16+ : \$680.16.9+ Ans.

13. By Table I. the amount of \$1 for 25 years, is \$47.727099; therefore \$800 will amount to $\$800\times\$47.727099=\$38181.67.9+$ Ans.

14. By Table I. the amount of \$1 for 30 years is \$79.058186; therefore the amount of \$40.00 is $\$40\times\$79.058186=\$3162.32.7+$ Ans.

15. By Table II. the present worth of an annuity of \$1 for 40 years is \$15.046297; therefore the present worth of \$500 is $\$500\times\$15.046297=\$7523.14.8+$ Ans.

16. By Table I. the amount of \$1 for 6 years is \$6.975319; the amount, therefore, of \$700, is $\$700\times\$6.975319=\$4882.72.3+$ Ans.

SECTION LX.

ALLIGATION.

(p. 221.)

2.

19bu. at $1.00 = $19.00
40bu. at .66 = 26.40
11bu. at .50 = 5.50

70bu. : $50.90 : : 1bu. : $0.72.7$\frac{1}{7}$ Ans.

3.

3lb. at 22 carats = 66 carats,
3lb. at 20 carats = 60 carats.

6lb. : 126 carats : : 1lb. : 21 carats, Ans.

4.

20lb. at $0.70 = $14.00
15lb. at .60 = 9.00
80lb. at .40 = 32.00

115lb. : $55.00 : : 1lb. : 0.47\frac{19}{23}$ Ans.

6.

	$ cts.	lb.	lb.	
Mean rate $0.80	.50	20	20 at $0.50	Ans.
	.75	20	20 at .75	
	1.00	30 + 5	35 at 1.00	

7.

	cts.		
Mean rate $0.20	.15	4oz. of 15 carats,	Ans.
	.19	3oz. of 19 carats,	
	.23	1oz. of 23 carats,	
	.24	5oz. of 24 carats,	

8.

	$ cts.		
Mean rate $0.75	0	45gal. of water at $0.00	Ans.
	.60	5gal. of wine at .60	
	.80	15gal. of wine at .80	
	1.20	75gal. of wine at 1.20	

10.

Mean rate 8s.	12s.	4lb.
	10s.	2lb.
	6s.	2lb.
	4s.	4lb.

4lb. : 2lb. : : 20lb. : 10lb. at 6s.
4lb. : 2lb. : : 20lb. : 10lb. at 10s. } Ans
4lb. : 4lb. : : 20lb. : 20lb. at 12s.

11.

Mean rate $1.00	$1.75	100	100gal.
	1.25	100	100gal.
	.00	75+25	100gal.

100gal. : 100gal. : : 20gal. : 20gal. port wine,
100gal. : 100gal. : : 20gal. : 20gal. temperance wine, } Ans.

13.

	cts.		
Mean rate $0.15	.08	5	5lb.
	.10	5	5lb.
	.12	5	5lb.
	.20	7 + 5 + 3	15lb.
			30lb.

lb.	lb.	lb.	lb.		
30 :	200 : :	5 :	$33\frac{1}{3}$	at .08cts.	Ans.
30 :	200 : :	5 :	$33\frac{1}{3}$	at .10cts.	
30 :	200 : :	5 :	$33\frac{1}{3}$	at .12cts.	
30 :	200 : :	15 :	100	at .20cts.	

14.

$$\text{Mean rate 20 carats,}\left\{\begin{array}{l} \text{carats.} \\ 15 \\ 17 \\ 18 \\ 22 \end{array}\right. \begin{array}{l} \\ 2 \\ 2 \\ 2 \\ 5+3+2 \end{array} \left|\begin{array}{r} \\ 2\text{oz.} \\ 2\text{oz.} \\ 2\text{oz.} \\ 10\text{oz.} \\ \hline 16\text{oz.} \end{array}\right.$$

$$\left.\begin{array}{l} \text{oz.}\quad \text{oz.}\quad\ \ \text{oz.}\quad \text{oz.} \\ 2 : \ 2 :: 5 : \ 5 \text{ of } 15 \text{ carats,} \\ 2 : \ 2 :: 5 : \ 5 \text{ of } 17 \text{ carats,} \\ 2 : 10 :: 5 : 25 \text{ of } 22 \text{ carats,} \end{array}\right\} \text{Ans.}$$

SECTION LXI.

PERMUTATIONS AND COMBINATIONS.

2. (p. 225.) 1×2×3×4×5×6×7×8×9×10= 3628800 days, Ans.

3. 1×2×3×4×5×6×7×8×9×10×11×12= 479001600 changes. 365da. 5h. 49m. = 525949 minutes. 479001600÷10=47900160 minutes. 47900160÷525949 = 91y. 38801m. = 91y. 26da. 22h. 41m. Ans.

4. 1×2×3×4×5×6×7×8×9×10×11×12×13× 14×15×16×17×18×19×20×21×22×23×24×25× 26 = 403291461126605635584000000 changes, Ans.

6. 20×19×18×17×16×15×14×13×12×11 = 670442572800; 1×2×3×4×5×6×7×8×9×10= 3628800; 670442572800 ÷ 3628800 = $1847.56 Ans.

In performing this 6th question, we adopt the following

RULE.

Take the series 1, 2, 3, &c. up to the number to be taken at a time, and find the product of all the terms. Then take *a series of as many terms, decreasing by 1, from the given number, out of which the election is to be made, and find their product. Then divide the last product by the former, and the quotient is the number sought.*

7. $26 \times 25 \times 24 \times 23 \times 22 \times 21 = 165765600$ words, Ans.

9. $6 \times 8 \times 9 \times 9 = 3888$ choices, Ans.

10. $6 \times 6 \times 6 \times 6 \times 6 = 7776$ changes, Ans.

SECTION LXII.

SINGLE POSITION.

2. (p. 227.) By Analysis. — If $\frac{1}{3}$ and $\frac{1}{4}$ of any number be taken, $\frac{5}{12}$ will remain; and if 60 be $\frac{5}{12}$ of a number, 12 will be $\frac{1}{12}$; and if 12 be $\frac{1}{12}$, 12 twelfths will be 144 Ans.

3. By Analysis. — If any number be increased $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of itself, the number will be $2\frac{1}{12}$, or $\frac{25}{12}$ of itself; and if $\frac{25}{12}$ of a number be 125, $\frac{1}{12}$ will be 5, and $\frac{12}{12}$ will be 60 Ans.

4. By Analysis. — B's age is $\frac{1}{2}$ of A's, and C's age is $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$ of A's. These fractions, then, $\frac{1}{1}$, $\frac{1}{2}$, and $\frac{1}{6}$, will express the relative proportion of their ages. The sum of these numbers is $\frac{1}{1} + \frac{1}{2} + \frac{1}{6} = \frac{5}{3}$. If $\frac{5}{3}$ be 140, $\frac{3}{3}$ will be $84 =$ A's age; B's age will therefore be $\frac{1}{2} \times 84 = 42$; C's age will be $\frac{1}{3}$ of $42 = 12$ Ans.

5. By Analysis. — Any sum, at 6 per cent. simple interest, will, in 10 years, amount to $1\frac{3}{5}$ of itself; thus, $\$100 \times .06 = \$6$; $6 \times 10 = \$60$; $60 + 100 = \$160$; $\frac{160}{100} = 1\frac{3}{5} = \frac{8}{5}$; and if \$560 be $\frac{8}{5}$, \$70 will be $\frac{1}{5}$, and $\frac{5}{5}$ will be \$350 Ans.

6. By Analysis. — Seven-eighths of any number is $\frac{27}{40}$ more than $\frac{1}{5}$ of it; thus, $\frac{7}{8}-\frac{1}{5}=\frac{27}{40}$; and if $\frac{27}{40}$ be 81, $\frac{1}{40}$ will be $\frac{81}{27}$, and $\frac{40}{40}$ will be $\frac{81}{27}\times\frac{40}{1}=\frac{3240}{27}=120$ Ans.

7. By Analysis. — The difference between $\frac{3}{5}$ and $\frac{4}{7}$ is $\frac{1}{35}$; and if $\frac{1}{35}$ of a number be $2\frac{17}{35}=\frac{87}{35}$, the whole number, or $\frac{35}{35}$, will be $\frac{87}{35}\times\frac{35}{1}=87$ Ans.

DOUBLE POSITION.

2. (p. 229.) By Analysis. — A having gained a sum equal to $\frac{1}{4}$ of his stock, his present stock will be $\frac{5}{4}$ of his original stock. B having lost $225, and his present stock being $\frac{1}{2}$ of A's, that is, $\frac{1}{2}$ of $\frac{5}{4}=\frac{5}{8}$ of what he had at first; it will therefore follow, that $225 is $\frac{8}{8}-\frac{5}{8}=\frac{3}{8}$ of B's original stock. And if $\frac{3}{8}$ be $225, $\frac{1}{8}$ is $75; and if $\frac{1}{8}$ be $75, $\frac{8}{8}$, or the whole stock, will be 8 × $75 = $600.00 Ans.

3. By Analysis. — It appears that his oldest son was 12 years older than the youngest, and that the oldest was also twice as old as the youngest; the oldest must therefore be 24 years old, and the youngest 12; the second son being 4 years younger than the first, and the third being 4-years older than the youngest, their ages will be 20 and 16 Ans.

4. By Analysis. — It appears by the question, that, if the saddle be put on the first horse, the horse and saddle will be twice the value of the second horse; that is, the first horse will be equal to twice the value of the second horse minus the saddle. Again, if the saddle be put on the second horse, the second horse and the saddle will be equal to three times the value of the first horse; that is, three times the value of the first horse is equal to the value of the second horse plus the saddle. We have before shown, that the value of the first horse is equal to twice the value of the second horse minus the saddle; therefore, by adding these quantities, we find that four times the value of the first

horse is equal to three times the value of the second horse. Again, as the first horse and saddle were worth twice the second horse, it will follow, that the saddle is worth twice the second horse minus the first horse; that is, the saddle is worth twice the second horse minus three-fourths the second horse, because we have before shown, that the first horse is three-fourths the value of the second horse; therefore the saddle is worth five-fourths of the second horse; but the saddle is worth $50; and if five-fourths of the second horse is worth $50, one-fourth is worth $10, and four-fourths, that is, the whole value of the second horse, will be $40; and as we have before shown, that the first horse is worth three-fourths of the second horse, his value will therefore be $30 Ans.

5. By Analysis. — Reducing $\frac{2}{3}$ and $\frac{8}{33}$ to a common denominator, the fractions will be $\frac{22}{33}$ and $\frac{8}{33}$; therefore $\frac{22}{33}$ of the time past from noon will be equal to $\frac{8}{22}$ of the time to midnight. Divide 12 hours into two parts, in the proportion of 8 to 22, and the first part will be the time from noon; thus, $8 + 22 = 30 : 8 :: 12$h. : 3h. 5m. Ans.

6. By Analysis. — If A saves $\frac{1}{12}$ of his income, he spends $\frac{11}{12}$ of it. If B, in 10 years, spends $600 more than his income, he must expend each year $60 more than his income. And as B spends $100 yearly more than A, it is evident A will save $40 per annum; and as A spends $\frac{11}{12}$, $40 must be $\frac{1}{12}$ of his income; therefore his whole income will be 12 times $40 = $480 Ans.

7. By the Rule of Position. — It appears by the tenor of the question, that 90 must be so divided into two parts, that, if the larger part be multiplied by 60, and the smaller part by 80, the difference of the products shall be 3300. We therefore suppose the larger part to be 60, and the smaller part 30, and proceed according to the Rule; thus, $60 \times 60 = 3600$; $30 \times 80 = 2400$. The difference between these products is $3600 - 2400 = 1200$, which is

3300 — 1200 = 2100 *less* than it should be. Again, we divide 90 into the two parts, 80 and 10, and multiply them by 60 and 80 as before, and find the result as follows; 80 × 60 = 4800; 10 × 80 = 800; 4800 — 800 = 4000, which is 4000 — 3300 = 700 *more* than it should be. We then proceed by the Rule, thus; 60 × 700 = 42000; 80 × 2100 = 168000; 42000 + 168000 = 210000 sum of the products; 2100 + 700 = 2800 sum of the errors; 210000 ÷ 2800 = 75 = larger number; and 90 — 75 = 15 smaller number. Therefore we find he labored 75 days, and was idle 15 days, Ans.

9. As the head and tail of the fish weigh 30lb. and $\frac{1}{6}$ of the body; and as the body weighs as much as the head and tail, it is evident, that 30lb. is $\frac{5}{6}$ of half the weight of the fish, and 30lb. is $\frac{5}{6}$ of 36lb.; therefore the head and tail weigh 36lb. The body also must weigh 36lb., because, by the question, it weighs as much as the head and tail both; therefore the whole weight of the fish will be 36 + 36 = 72lb. Ans.

10. The weights must be 1, 3, 9, 27, and 81lb., and these numbers may be obtained by multiplying; thus, 1 × 1 = 1, 1 × 3 = 3, 3 × 3 = 9, 9 × 3 = 27, 27 × 3 = 81 Ans.

SECTION LXV.

GEOMETRY.

1 (p. 257.) 15 × 2 = 30 feet, Ans.

2 128 × 48 = 6144 miles, Ans.

3. 693 × 693 = 480249 feet; 480249 ÷ 272$\frac{1}{4}$ = 1764p.; 1764 ÷ 160 = 11A. 0R. 4p. Ans.

4. $40 \times 40 = 1600$ feet; $20 \times 20 \times 2 = 800$ feet; $1600 - 800 = 800$ feet, Ans.

5. $\sqrt{3600} = 60$ feet; $3600 \div 2 = 1800$ feet; $\sqrt{1800} = 42.427 +$ feet; 60 feet $-$ 42.427 feet $=$ 17.573 feet; $17.573 \div 2 = 8.78 +$ feet, Ans.

6. $8 \times 12 = 96$ feet, Ans.

7. $12 \times 18 = 216$ feet; $216 \div 2 = 108$ feet, Ans.

8. $15.6 + 9.2 + 10.4 = 35.2$ feet; $35.2 \div 2 = 17.6$ feet; $17.6 - 15.6 = 2.00$; $17.6 - 9.2 = 8.4$; $17.6 - 10.4 = 7.2$; $17.6 \times 2 \times 8.4 \times 7.2 = 2128.896$; $\sqrt{2128.896} = 46.139 +$ feet, Ans.

9. $144 \times 3.141592 = 452.389248$ feet, Ans.

10. $7964 \times 3.141592 = 25019.638688 +$ miles, Ans.

11. $50 \times .886227 = 44.31135 +$ rods, Ans.

12. $80 \times .707016 = 56.56128 +$ feet, Ans.

13. $761 \times 761 \times .785398 = 454840.475158$ feet, Ans.

14. $320 \times 3 = 960 \times 960 \times .785398 = 7238.227968$ rods; $7238.227968 \div 160 = 4523.89 +$ acres, Ans.

16. $360 \times .282094 = 101.55 +$ rods, Ans.

17. $5000 \times .2756646 = 1378.323 +$ feet, Ans.

18. $5000 \times .225079 = 1125.395$ rods, Ans.

19. $1\frac{1}{2} \times 1\frac{1}{2} \times 1\frac{1}{2} = 3\frac{3}{8}$ feet, Ans.

20. $6 \times 2\frac{1}{2} \times 1\frac{3}{4} = 26\frac{1}{4}$ feet, Ans.

21. $3\frac{1}{6} \times 2\frac{2}{3} \times 2\frac{1}{2} = 21\frac{1}{9}$ feet, Ans.

22. $2\frac{1}{2} \times 3 = 7\frac{1}{2}$; $7\frac{1}{2} \div 2 = 3\frac{3}{4}$; $3\frac{3}{4} - 2\frac{1}{2} = 1\frac{1}{4} = \frac{5}{4}$; $3\frac{3}{4} = \frac{15}{4}$; $\frac{15}{4} \times \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} = \frac{1875}{256} = 7.32421875$ feet; $\sqrt{7.32421875} = 2.7063 +$ feet; $2.7063 \times 12 = 32.47 +$ feet, Ans.

23. $5 + 4 + 3 = 12$; $12 \div 2 = 6$ feet; $6 - 5 = 1$; $6 - 4 = 2$; $6 - 3 = 3$; $6 \times 1 \times 2 \times 3 = 36$; $\sqrt{36} = 6$; $6 \times 10 = 60$ feet, Ans.

24. $2\frac{1}{2} \times 2\frac{1}{2} \times .785398 \times 12\frac{1}{2} = 61.35921875$; $61.35921875 \div 3 = 20.45 +$ feet, Ans.

25. $5 + 6 + 7 = 18$; $18 \div 2 = 9$; $9 - 5 = 4$; $9 - 6 = 3$; $9 - 7 = 2$; $9 \times 4 \times 3 \times 2 = 216$; $\sqrt{216} = 14.69693 +$;

$14.69693 \times 14\frac{1}{2} = 213.105485$; $213.105485 \div 3 = 71.035+$ feet, Ans.

26. $75 \div .785398 = 95.492987$; $95.492987 \div 2 = 47.746493+$ square rods, Ans.

27. $75 \div .785398 = 95.492987$; $\sqrt{95.492987} = 9.772+$ rods, Ans.

28. $25 \times 25 = 625$; $625 \div 2 = 312.5$; $625 - 312.5 = 312.5$; $25 \times 25 \times .785398 = 490.87375$; $625 - 490.87375 = 134.12625$; $490.87375 - 312.5 = 178.37375$ Ans.

29. $9 \times 20 = 180$; $180 \div 2 = 90$ feet, Ans.

30. $24 \times 12 = 288$; $12 \times 12 = 144$; $144 \div 3 = 48$; $288 + 48 = 336$; $336 \times .785398 \times 40 = 10555.74912$; $10555.74912 \div 144 = 73.303 +$ feet, Ans.

31. $12 \times 12 \times 12 \times .5236 = 904.78$ inches, Ans.

32. $25000 \times .31831 = 7957.75$; $7957.75 \times 7957.75 = 63325785.0625$; $63325785.0625 \times 7957.75 = 503930766081.109375$; $503930766081.109375 \times .5236 = 263858149120.06886875$ cubic miles, Ans.

33. $24 \times 3.141592 = 75.398208$; $75.398208 \times 24 = 1809.556992$ inches, Ans.

34. $7957.75 \times 25000 = 198943750$ square miles, Ans.

36. $40 \times 40 = 1600$; $1600 \div 3 = 533.3333+$; $\sqrt{533.3333} = 23.09 +$ inches, Ans.

37. $20 \times 20 = 400$; $400 \div 3 = 133.3333+$; $\sqrt{133.3333} = 11.547 +$; $11.547 \times 11.547 \times 11.547 = 1539.6 +$ inches, Ans.

38. 10ft. = 120in.; 5ft. = 60in.; 4ft. = 48in.; $120 \times 60 \times 48 = 345600$; $345600 \div 231 = 1496\frac{24}{231}$ gal. Ans.

39. 12ft. = 144in.; 6ft. = 72in.; 2ft. = 24in.; $144 \times 72 \times 24 = 248832$in.; $248832 \div 282 = 882\frac{18}{47}$ gal. Ans.

40. 15ft. = 180in.; 5ft. = 60in.; 7ft. = 84in.; $180 \times 60 \times 84 = 907200$in.; $907200 \div 2150.42 = 421.8+$ bushels, Ans.

42. $40 - 30 = 10$; $10 \times .65 = 6.5$; $30 + 6.5 = 36.5$; $36.5 \times 36.5 = 1332.25$; $1332.25 \times 50 = 66612.5$; $66612.5 \div 359 = 185.55 +$ gal. Ans.

43. $40 \times 30 = 1200$; $40 - 30 = 10$; $10 \times 10 = 100$; $100 \div 3 = 33\frac{1}{3}$; $1200 + 33\frac{1}{3} = 1233\frac{1}{3}$; $1233\frac{1}{3} \times 50 = 61666.66+$; $61666.66 \div 294 = 209.7+$ gal. Ans.

SECTION LXVI.

TONNAGE OF VESSELS.

1. (p. 265.) $65 \times 20 \times 10 = 13000$; $13000 \div 95 = 136\frac{16}{19}$ tons, Ans.

2. $70 \times 24 \times 12 = 20160$; $20160 \div 95 = 212\frac{4}{19}$ tons, Ans.

3. $\frac{3}{5}$ of $30 = 18$; $70 - 18 = 52$; $52 \times 30 \times 9 = 14040$; $14040 \div 95 = 147\frac{15}{19}$ tons, Ans.

4. $\frac{3}{5}$ of $22 = 13.2$; $75 - 13.2 = 61.8$; $61.8 \times 22 \times 12 = 16315.2$; $16315.2 \div 95 = 171\frac{351}{475}$ tons, Ans.

5. $\frac{3}{5}$ of $35 = 21$; $98 - 21 = 77$; $77 \times 35 \times 17\frac{1}{2} = 47162.5$; $47162.5 \div 95 = 496\frac{17}{38}$ tons, Ans.

6. $\frac{3}{5}$ of $40 = 24$; $180 - 24 = 156$; $156 \times 40 \times 20 = 124800$; $124800 \div 95 = 1313\frac{13}{19}$ tons, Ans.

7. $\frac{3}{5}$ of $21 = 12.6$; $78 - 12.6 = 65.4$; $65.4 \times 21 \times 9 = 12360.6$; $12360.6 \div 95 = 130\frac{53}{475}$ tons, Ans.

8. $\frac{3}{5}$ of $30 = 18$; $159 - 18 = 141$; $141 \times 30 \times 15 = 63450$; $63450 \div 95 = 667\frac{17}{19}$ tons, Ans.

9. $\frac{3}{5}$ of $80 = 48$; $479 - 48 = 431$; $431 \times 80 \times 48 = 1655040$; $1655040 \div 95 = 17421\frac{9}{19}$ tons, Ans.

SECTION LXVII.

MENSURATION OF LUMBER.

1. (p. 257.) $24 \times 8 = 192$; $192 \div 12 = 16$ feet, Ans.

2. $30 \times 16 = 480$; $480 \div 12 = 40$ feet, Ans.

3. $3 \times 5 \times 15 \times 3 = 675$; $675 \div 12 = 56\frac{1}{4}$ feet, Ans.

4. $2 \times 6 \times 10 \times 20 = 2400$; $2400 \div 12 = 200$ feet, Ans.

5. $40 \div 4 = 10$; $10 \times 10 = 100$; $100 \times 30 = 3000$; $3000 \div 144 = 20\frac{5}{6}$ feet, Ans.

6. $56 \div 4 = 14$; $14 \times 14 = 196$; $196 \times 50 = 9800$; $9800 \div 144 = 68\frac{1}{18}$ feet, Ans.

7. $120 \div 4 = 30$; $30 \times 30 = 900$; $900 \times 90 = 81000$; $81000 \div 144 = 562\frac{1}{2}$ feet, Ans.

SECTION LXVIII.

PHILOSOPHICAL PROBLEMS.

2. (p. 267.) $1 \times 1 : \frac{1}{4} \times \frac{1}{4} :: 39.2 : 2\frac{9}{20}$in. Ans.

3. $1 \times 1 : 60 \times 60 :: 39.2 : 141120$in.; $141120 \div 12 = 11760$; $11760 \div 3 = 3920$yd. Ans.

4. $39.2 : 100 \times 12 :: 1 \times 1 : 30.6122$; $\sqrt{30.6122} = 5.53$sec. Ans.

5. $2000 + 4000 = 6000$; $6000^2 : 4000^2 :: 900$lb : 400lb. Ans.

6. $20000 + 4000 = 24000$; $24000^2 : 4000^2 :: 144$lb. : 4lb. Ans.

7. $1 : 2 :: 4000^2 : 32000000$; $\sqrt{32000000} = 5656.85$; $5656.85 - 4000 = 1656.85$ miles, Ans.

8. $4000^2 : 240000^2 :: 150$lb. : 540000lb. Ans.

9. $400 : 900 :: 4000^2 : 36000000$; $\sqrt{36000000} = 6000$; $6000 - 4000 = 2000$ miles, Ans.

10. $2180^3 = 1036023200$; $1036023200 \times 494 = 511795460800$ = cube of the moon's diameter multiplied by its density; $7964^3 = 505139057344$; $505139057344 \times 400 = 202055622937600$ = cube of the earth's diameter multiplied by its density. $511795460800 : 202055622937600 :: 6 : 236.8+$feet, Ans.

11. $2654 \times 400 = 1061600 = \frac{2}{3}$ of the earth's semidiameter multiplied by its density;
$294415 \times 100 = 29441500 = \frac{2}{3}$ of the sun's semidiameter multiplied by its density;
$26347 \times 67 = 1765249 = \frac{2}{3}$ of Saturn's diameter multiplied by his density;
$29723 \times 94.5 = 28088235 = \frac{2}{3}$ of Jupiter's diameter multiplied by his density;
$726 \times 494 = 358644 = \frac{2}{3}$ moon's diameter multiplied by its density.

$$\left.\begin{array}{llll} 1061600 : 29441500 & :: 170\text{lb.} : & 4714.6+\text{lb.} = & \text{the man's weight at the sun;} \\ 1061600 : 1765249 & :: 170\text{lb.} : & 282.6+\text{lb.} = & \text{the man's weight at Saturn;} \\ 1061600 : 28088235 & :: 170\text{lb.} : & 449.7+\text{lb.} = & \text{the man's weight at Jupiter;} \\ 1061600 : 358644 & :: 170\text{lb.} : & 57.4+\text{lb.} = & \text{the man's weight at the moon;} \end{array}\right\}\text{Ans.}$$

13. $1^2 : 60^2 = 3600 :: 16$ft. : 10m. 1600yd. Ans.

14. $1^2 : 3600^2 = 12960000 :: 16$ft. : 39272m. 1280yd. Ans.

15. $24 \times 60 \times 60 \times 9 = 777600$ = seconds in 9 days; $1^2 : 777600^2 = 604661760000 :: 16$ft. : 9674588160000ft.; $9674588160000 \div 3 = 3224862720000$yd.; $3224862720000 \div 1760 = 1832308363$m. 1120yd. Ans.

16. 660 ÷ 8 = 82½ ; 82½ × 82½ = 6806¼ft. Ans.

17. 1000 ÷ 8 = 125 ; 125 × 125 = 15625ft. ; 15625 ÷ 5280 = 2m. 5065ft. Ans.

18. 200 ÷ 8 = 25 ; 25 ÷ 4 = 6¼sec. Ans.

19. 320 ÷ 8 = 40 ; 40 ÷ 4 = 10sec. Ans.

20. √40000 = 200 ; 200 ÷ 4 = 50sec. Ans.

21. √400 = 20 ; 20 ÷ 4 = 5sec. Ans.

23. 64 × 64 = 4096 ; √4096 = 64 ; 64 × 2240 × 4 = 573440lb. Ans.

SECTION LXIX.

MECHANICAL POWERS.

1. (p. 272.) 1lb. : 9lb. : : 170lb. : 1530lb. Ans.

2. 9lb. : 1lb. : : 1530lb. : 170lb. Ans.

3. 170lb. : 1530lb. : : 1lb. : 9lb. Ans.

4. 1530lb. : 170lb. : : 9lb. : 1lb. Ans.

5. 4.000.000.000.000.000.000.000 × 100 = 400.000.000.000.000.000.000.000 ; 200lb. : 400.000.000.000.000.000.000.000lb. : : 6000 miles : 12.000.000.000.000.000.000.000.000 miles, Ans.

6. 6000m. : 12.000.000.000.000.000.000.000.000m. : : 1in. : 2.000.000.000.000.000.000.000in. ; 2.000.000.000.000.000.000.000 ÷ 12 = 166.666.666.666.666.666.666⅔ft. ; 100ft. : 166.666.666.666.666.666.666 : : 1sec. : 1.666.666.666.666.666.666⅔sec. ; 1.666.666.666.666.666.666 ÷ 60 = 27.777.777.777.777.777m. 46⅔sec. ; 27.777.777.777.777.777 ÷ 60 = 462.962.962.962.962h. 57m. ; 462.962.962.962.962 ÷ 8766 = 52.813.479.690y. 17da. 14h. 57m. 46⅔sec. Ans.

7. 4ft. = 48in. : 6in. : : 960lb. : 120lb. Ans.

8. 6in. : 48in. : : 120lb. : 960lb. Ans.

9. 120lb. : 960lb. : : 6in. : 48in. = 4ft. Ans.

10. 960lb. : 120lb. : : 48in, : 6in. Ans.

11. 400lb. ÷ 2 = 200lb. Ans.

12. 2 × 3 × 10 = 60lb. Ans.

13. 2 × 2 × 144 = 576lb. Ans.

14. 6in. : 60in. : : 1 : 10; 144lb. × 2 × 2 × 10 = 5760lb. Ans.

15. 1000lb. × 10 = 10.000; 10.000 ÷ 50 = 200lb. Ans.

16. 1728lb. × 5 = 8640; 8640 ÷ 600 = 14$\frac{2}{5}$lb. Ans.

17. 20000lb. × 50ft. = 1000000; 1000000 ÷ 5280 = 189$\frac{13}{33}$lb. Ans.

18. 1 : 3.141592in. × 10 × 2 × 12 : : 100lb. : 75398.208 + lb. Ans.

19. 3.141592 × 2 × 100 : $\frac{1}{2}$in. : : 100000lb. : 79.5774 + lb. Ans.

20. $\frac{1}{2}$in. : 3.141592in. × 200 : : 79.5774 + lb. : 100.000lb. Ans.

21. 100.000lb. : 79.5774 + lb. : : 3.141592in. × 2 × 100 : $\frac{1}{2}$in. Ans.

22. 79.5774 + lb. : 100.000lb. : : $\frac{1}{2}$in. : 628.3184 + in.; 628.3184 ÷ 3.141592 = 200in.; 200 ÷ 2 = 100in. Ans.

23. 2in. : 20in. : : 100lb. : 1000lb. Ans.

24. .75in. : 3.141592 × 16 × 2 × 12 : : 200lb. : 3216990.208lb.; 1.5in. : 12in. : : 3216990.208lb. : 2573592.116lb. Ans.

SECTION LXX.

SPECIFIC GRAVITY.

1. (p. 277.) 10lb.—6$\frac{1}{6}$lb. = 3$\frac{5}{6}$lb.; 3$\frac{5}{6}$lb. : 10lb. : : 1000oz. : 2608.6 + oz. Ans.

2. 18lb.—16lb. = 2lb.; 15lb. + 18lb. = 33lb.; 33lb.—6lb. = 27lb.; 27lb.—2lb. = 25lb.; 25lb. : 15lb. : : 1000oz. : 600oz. Ans.

SECTION LXXI.

ASTRONOMICAL PROBLEMS.

3. (p. 278.) $1841 \div 4 = 460$; $1841 + 460 = 2301$; $2301 \div 7 = 328$, and 5 remainder; $8 - 5 = 3 = C$, Ans.

4. $1899 \div 4 = 474$; $1899 + 474 = 2373$; $2373 \div 7 = 339$, and 0 remainder; therefore A, Ans.

5. $1896 \div 4 = 474$; $1896 + 474 = 2370$; $2370 \div 7 = 338$, and 4 remainder; $8 - 4 = 4 = D$ and E, Ans.

6. $1786 \div 4 = 446$; $1786 + 446 = 2232$; $2232 \div 7 = 318$, and 6 remainder; $7 - 6 = 1 = A$, Ans.

7. $1837 \div 4 = 459$; $1837 + 459 = 2296$; $2296 \div 7 = 328$, remainder 0; therefore A, Ans.

MISCELLANEOUS QUESTIONS

1. (p. 279.) $6\frac{2}{3} \div 7\frac{3}{5} = \frac{50}{57}$ Ans.
2. $4\frac{1}{2} \times 2 = 9$; $\sqrt{9} = 3$ Ans.
3. $11\frac{3}{7} \times 5 = 57\frac{1}{7}$ Ans.
4. $9\frac{4}{7} \div 7\frac{3}{4} = 1\frac{51}{217}$ Ans.
5. $\frac{7}{19\frac{3}{5}} = \frac{35}{98} = \frac{5}{14}$ Ans.
6. $\frac{4}{7}$ of a ton = 11cwt. 1qr. 20lb.; $\frac{9}{10}$ of a cwt. = 3qr. $16\frac{4}{5}$lb.; 11cwt. 1qr. 20lb. + 3qr. $16\frac{4}{5}$lb. = 12cwt. 1qr. $8\frac{4}{5}$lb. Ans.
7. $360° : 1° :: 23$h. 56m. 3sec. : 3h. 59m. 20 + sec. Ans.
8. $\frac{2}{7} \times \frac{5}{9} = \frac{10}{63}$; $\frac{1}{1} - \frac{10}{63} = \frac{53}{63}$ Ans.
9. $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$; $\frac{1}{3} = \frac{4}{12}$; $\frac{1}{4} = \frac{3}{12}$;

$$\left.\begin{array}{l} \frac{7}{12} : \frac{4}{12} :: \$100.000 : \$57142\frac{6}{7} \text{ A's part,} \\ \frac{7}{12} : \frac{3}{12} :: \$100.000 : \$42857\frac{1}{7} \text{ B's part,} \end{array}\right\} \text{Ans.}$$

10. $\frac{12}{13} \times \frac{6}{1} = \frac{72}{13} =$ 5h. 32m. $18\frac{6}{13}$sec. Ans.

NOTE.—We use the fraction $\frac{12}{13}$, because the minute-hand passes the hour-hand twelve times in thirteen hours.

11. As there is a son and a daughter, the son will have $\frac{4}{7}$ of the estate, the wife $\frac{2}{7}$, and the daughter $\frac{1}{7}$. If there had been only a daughter, her share would have been $\frac{2}{3}$; consequently she loses $\frac{2}{3} - \frac{2}{7} = \frac{8}{21}$. Hence

$\frac{8}{21} : \frac{7}{21} ::$ \$2400 : \$2100 Ans.

12. If the first man's share be subtracted from the whole, there will remain $\frac{18}{18} - \frac{7}{18} = \frac{11}{18}$; and $\frac{7}{18}$ of $\frac{11}{18} = \frac{77}{324} =$ the second son's share. And $\frac{7}{18} - \frac{77}{324} = \frac{49}{324} =$ difference of their legacies. $\frac{7}{18} = \frac{126}{324}$; $\frac{126}{324} + \frac{77}{324} = \frac{203}{324} =$ legacy of both sons. Hence $\frac{324}{324} - \frac{203}{324} = \frac{121}{324} =$ wife's legacy. Therefore

$\frac{49}{324} : \frac{121}{324} ::$ 257£. 3s. 4d. : 635£. 0s. $10\frac{30}{49}$d. Ans.

13. $63 \times 12 \times 12 \times 1000 \times 3 = 27216000$; $27216000 \div 16 = 1701000$lb.; $1701000 \div 2240 = 759\frac{3}{8}$ tons, Ans.

14. 4ft. = 48in.; 6in. × 2 = 12in.; 48 — 12 = 36in.; 36 ÷ 2 = 18in.; 18 + 12 = 30in.; 48in. : 18in. : : 200lb. : 75lb. Ans. 48 — 30 = 18in.; 48in. : 18in. : : 200lb. : 75lb. Ans.

15. To perform this question, we first find how long each pillar must be to terminate in a cone. We then find the contents of the whole cone, from which we subtract the part that is applied to complete the cone, and the remainder is the contents of each pillar. 25ft. 4in. = 304in.; 4ft. 5in. = 53in.; 3ft. 5in. = 41in.; 53 — 41 = 12in. = difference of the diameters of the ends of the pillars. 12in. : 53in. : : 304in. : 16112in. = $1342\frac{2}{3}$ft. = length of the larger cone; $1342\frac{2}{3} - 304 = 1038\frac{2}{3}$in. = length of the smaller cone. $53 \times 53 \times .785398 = 2206.182982$in. = area of the base of the larger cone. $2206.182982 \times 1342\frac{2}{3} =$ 2962168.350498; $2962168.350498 \div 3 = 987389.450166$ = contents of the larger cone; $987389.450166 \times 8 =$

7899115.601328 = contents of 8 cones; 41×41×.785398 = 1320.254038 = area of the base of the smaller cone; 1320.254038×1038$\frac{2}{3}$ = 1371303.860802; 1371303.860802 ÷ 3 = 457101.286934 = contents of the smaller cone; 457101.286934 × 8 = 3656810.295472 = contents of the 8 smaller cones. We now proceed to subtract the contents of the 8 smaller cones from the contents of the 8 larger cones, and the remainder will be the contents of the pillars required by the question. 7899115.601328 — 3656810.295472 = 4242305.305856in. = contents in cubic inches of the pillars; 4242305.305856 ÷ 1728 = 2455.037792 = cubic feet in the pillars, Ans. 2455.037792×3000 = 7365113.376 = weight in ounces; 7365113.376 ÷ 16 = 460319.586 pounds; 460319.586 ÷ 2240 = 205.49981 + tons, Ans.

16. If $\frac{3}{7}$ of a certain sum be taken, and $410 be left, it is evident that $410 is $\frac{4}{7}$ of that sum, which is $\overline{410\times7}\div4=$ 717\frac{1}{2}$. Now, if 717\frac{1}{2}$ remain of a certain quantity after $\frac{1}{4}$ be subtracted, it is certain that the number from which it is taken is $\frac{4}{3}$ of 717\frac{1}{2}$ = 956.66\frac{2}{3}$ Ans.

17. $\frac{1}{4}$: $15.60 :: $100 : $6240 = sum remitted; $96 : $100 :: $6240 : $6500 = value of goods sold; $6500 — $6240 = $260 = commission, Ans.

18. $107.50 : $100 :: $9675 : $9000; $\frac{9}{40}$ × $9000 = 2025£. sterling = the bill; $100 — $0.25 = $99.75;

$100 : $99.75 :: $9675 : 9650.81\frac{1}{4}$;

$102 : $100 :: 9650.81\frac{1}{4}$: 9461.58\frac{3}{34}$ for investment, Ans.

19. The same principle is adopted to perform this question, as was used in the operation of the 15th. As the monument, at the height of 220 feet, has half the diameter at the top, that it has at its base, it is evident, that, if it were raised to 440 feet, it would terminate in a point. The cylindrical avenue in the centre of the monument, being 15 feet diameter at the base, and 11 feet at the height of 220 feet, must be 825 feet in length before it terminates in

a point, as may be seen in the following operation; 15 — 11 = 4ft.; 4ft. : 15ft. :: 220ft. : 825ft. length of the cylinder, terminated in a cone; 15 × 15 × .785398 = 176.71455ft.; 176.71455 × 825 = 145789.50375ft.; 145789.50375 ÷ 3 = 48596.50125ft. contents of the whole cone; 825ft. — 220ft. = 605ft. = length of the cone above the monument; 11 × 11 × .785398 = 950.33158ft.; 950.33158 × 605 = 57495.06059ft.; 57495.06059 ÷ 3 = 19165.020196ft. contents of the small cone; 48596.50125 — 19165.020196 = 29431.481054ft. contents of the frustrum of the cone within the monument; 30 × 30 × 440 = 396000ft.; 396000 ÷ 3 = 132000ft. contents of the monument raised to a point at the height of 440ft.; 15 × 15 × 220 = 49500ft.; 49500 ÷ 3 = 16500ft. = contents of the square pyramid above the monument; 132000 — 16500 = 115500ft. contents of the monument without deducting the cylinder; 115500ft. — 29431.481054ft. = 86068.518946 cubic feet of the monument, that *is to be*, Ans.

20. $\frac{4}{12} \times 1 + \frac{4}{12} \times \frac{1}{2} + \frac{4}{12} \times \frac{1}{3} = \frac{22}{36}$ A's product;
$\frac{4}{12} \times \frac{1}{2} + \frac{4}{12} \times \frac{1}{3} = \frac{10}{36}$ M's product;
$\frac{4}{12} \times \frac{1}{3} = \frac{4}{36}$ P's product;
$\frac{36}{36}$ sum of the products.

$\frac{36}{36}$: \$300 :: $\frac{22}{36}$: \$183.33$\frac{1}{3}$ = A pays,
$\frac{36}{36}$: \$300 :: $\frac{10}{36}$: \$ 83.33$\frac{1}{3}$ = M pays, } Ans.
$\frac{36}{36}$: \$300 :: $\frac{4}{36}$: \$ 33.33$\frac{1}{3}$ = P pays,

21. A 20 × \$132 = \$2640
B 25 × \$120 = \$3000   85 × \$6 = \$510
C 40 × \$100 = \$4000
85   \$9640

\$9640 : \$510 :: \$2640 : \$139$\frac{161}{241}$ A receives,
\$9640 : \$510 :: \$3000 : \$158$\frac{172}{241}$ B receives, } Ans.
\$9640 : \$510 :: \$4000 : \$211$\frac{149}{241}$ C receives,

22. $\$100 \times \$5.00 = \$500$ given for the flour;
$\$500 \times \$0.20 = \$100$ gained on the flour.

$\$600$

$\$100 \times \$0.03.0\frac{1}{2} = \$3.05$ bank interest of $\$100$ for 6 months;
$\$100 - \$3.05 = \$96.95 : \$100 :: \$600 : \$618\frac{1688}{1939}$;
$\$618\frac{1688}{1939} \div 100 = \$6.18\frac{1688}{1939}$ Ans.

23. $5\frac{1}{4} \times 1\frac{7}{8} = \frac{315}{32}$ square yards of broadcloth; $\frac{7}{8} - \overline{\frac{1}{4} \times \frac{1}{4}} = \frac{7}{8} - \frac{1}{16} = \frac{13}{16}$yd. = width of flannel after being shrunk; $\frac{315}{32} \div \frac{13}{16} = \frac{5040}{416} = 12\frac{3}{26}$yd. = length of flannel necessary to line the broadcloth if it had not shrunk in *length;* 19yd. : 20yd. :: $12\frac{3}{26}$yd. : $12\frac{186}{247}$yd. quantity necessary to buy, Ans.

24. $40 \times 2 = 80$ft.; $30 - 2 = 28$ft.; $28 \times 2 = 56$ft.; $80 + 56 = 136$ft.; $136 \times 20 = 2720$ cubic feet; $2720 \times 1728 = 4700160$ cubic inches in the walls of the building; $8 \times 4 \times 2 = 64$ cubic inches in a brick; $4700160 \div 64 = 73440$ brick, Ans.

25. As the roof projects 1ft. over the plate, it will be 42ft. square; $42 \div 2 = 21$ft.; $21 \times 21 = 441$ft.; $15 \times 15 = 225$ft.; $441 + 225 = 666$ft.; $\sqrt{25.8069}+$ feet, being the distance from the top of the house to the plate; $25.8069 \div 2 = 12.9034$ft.; $12.9034 \times 42 \times 4 = 2167.7712$ square feet in the roof; $40 + 40 = 80$ft.; $80 \times 2 = 160$ft.; $160 \times 20 = 3200$ft. required to cover the body of the house; $3200 + 2167.7712 = 5367.7712$ square feet of boards necessary to cover the house, Ans.

26. $18.5 \times 18.5 \times 18.5 \times 8 = 50653$; $\sqrt[3]{50653} = 37$in. wide; $8 \times 8 \times 8 \times 8 = 4096$; $\sqrt[3]{4096} = 16$in. deep, Ans.

27. As the metal is 1 inch thick, the diameter of the *inner* sphere is 3 inches; $5 \times 5 \times 5 \times .5236 \times \frac{129}{500} = 16.8861$lb. weight of the shell, if it were solid iron; $3 \times 3 \times 3 \times .5236 \times \frac{129}{500} = 3.6473976$lb. weight of the inner sphere, if it were iron; $16.8861 - 3.6473976 = 13.2387+$ lb. Ans.

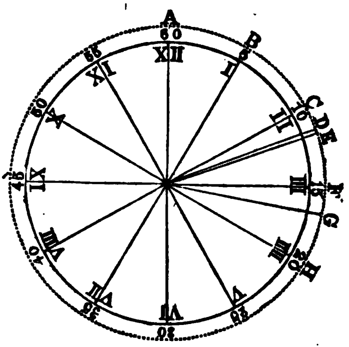
28. BY POSITION. — Suppose the time to be 16 minutes past 2 o'clock, the hour-hand must have passed $\frac{16}{60}$ of the dis-

tance from 2 o'clock to 3 o'clock; and if the minute-hand were in the place of the hour-hand, it would be 11m. 20sec. from 12 o'clock. And if the hour-hand were in the place of the minute-hand, the time would be 12 minutes past 3 o'clock. The difference between 12m. and 11m. 20sec. is 40sec. Let these be the first error. Again, suppose the time to be 18 minutes past 2 o'clock; the hour-hand, at that time, has passed $\frac{18}{60}$ of the distance from 2 o'clock to 3 o'clock; and if the minute-hand were in the place of the hour-hand, it would be 11m. 30sec. from 12 o'clock. And if the hour-hand were in the place of the minute-hand, the time would be 36 minutes past 3 o'clock. The difference between 36m. and 11m. 30sec. is 24m. 30sec. = 1470sec. Let these be the second error. Both errors are plus. We now proceed as in the operation:—

$$\begin{array}{rr}
16 & 40\,+ \\
18 & 1470\,+ \\
40 & 16 \\
\hline
720 & 23520 \\
 & -720 \\
\hline
\end{array}$$

$$\begin{array}{rl}
1470-40=1430\,) & 22800\;(\,15\text{m. }56\tfrac{92}{143}\text{sec. past 2 o'clock, A.} \\
 & 1430 \\
\hline
 & 8500 \\
 & 7150 \\
\hline
 & 1350 \\
 & 60 \\
\hline
1430\,) & 81000\;(\,56\tfrac{92}{143}\text{sec.} \\
 & 71500 \\
\hline
 & 9500 \\
 & 8580 \\
\hline
 & 920
\end{array}$$

BY ANALYSIS. — Let the annexed diagram represent the face of a clock; A, or 60, the 12 o'clock mark; B, 5 minutes; C, 10 minutes; F, 15 minutes; H, 20 minutes, &c. round the same. Suppose, then, the two hands to be together at A, and then the minute-hand to be moved forward $2\frac{1}{4}$ times round, and it will be at F, having moved 135 minutes; and the hour-hand will have moved to a point, which call D, between C and F. Suppose, then, the minute-hand to be moved still forward to the place required by the supposition, between F and H, which place call G. The hour-hand will then have moved to its place required between C and F, which place call E. Then, D E being $\frac{1}{12}$ of F G, and F G being $\frac{1}{12}$ of A E, it is evident that D E $= \frac{1}{12}$ of $\frac{1}{12}$ A E, that is, $\frac{1}{144}$ part of A E; and A D is the other $\frac{143}{144}$ of A E; and so, while the minute-hand moves from A twice round, and to G, the hour-hand moves from A to E; and while the minute-hand moves from A to E, the hour-hand moves from F to G; therefore, as 143 : 144 :: 135m. : $135\frac{135}{143}$m. $=$ 2h. $15\frac{135}{143}$m. $=$ the time at which the hands were in the first position, the minute-hand being $\frac{135}{143}$ of a minute beyond the 3 o'clock mark. Again, $135\frac{135}{143}$m. $\div 12 = 11\frac{47}{143}$m. $=$ the minute on the face where the hour-hand was at first. Again, $15\frac{135}{143}$m. $- 11\frac{47}{143}$m. $= 4\frac{88}{143}$m. $=$ the minutes between the two hands. Again, 60m. $- 4\frac{88}{143}$m. $= 55\frac{55}{143}$ $=$ the time between the hands changing places; and 2h. $15\frac{135}{143}$m. $+ 55\frac{55}{143}$m. $=$ 3h. $11\frac{47}{143}$m. $=$ the time on the clock when the hands would have changed places; and $11\frac{47}{143}$m. $\div 12 = \frac{135}{143}$ of 1 minute



= place of the hour-hand beyond the 3 o'clock mark at the time of changing places, Q. E. D.

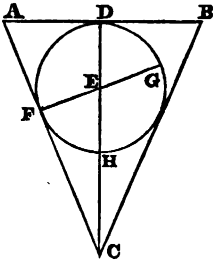
29. $20 \times 20 \times 20 = 8000$ cubic inches = contents of the larger cube. $20 \times 20 = 400$; $400 \div 3 = 133.33+$; $\sqrt{133.\dot{3}} = 11.5469+$; $\overline{11.5469^3}+ = 1539.58+$ cubic inches = contents of the smaller cube, Ans.

30. $90 \times 40 = 3600$; $\sqrt{3600} = 60$lb. true weight, Ans. $90 - 60 = 30$lb.; $60 - 40 = 20$lb.; 30lb. : 20lb. : : 3ft. : 2ft.; that is, the arms of the scales are to each other as 2ft. to 3ft. Ans.

31. Both wheels being of the same height, and the outer wheel making two turns, while the inner one makes only one turn, it will follow, that the outer ring will be twice the diameter of the inner ring. The distance between the rings being 5 feet; and the circumferences of circles being as their diameters, it will also follow, that the diameter of the inner ring will be 10 feet, and the diameter of the outer ring 20 feet. And if the diameter be 20 feet, the circumference will be $62.83+$ feet, Ans.

32. $72 \times 72 \times 3.141598 = 57001$d. = 237£. 10s. 1d. Ans.

33. The annexed diagram may represent the conical glass, A B C being the cone, and F D G H a globe or sphere immersed in it. If A B be 5 inches, A D will be 2.5 inches, because A D is half of A B. A D C is a right-angled triangle; therefore the side A C may be found; thus, $\sqrt{AD^2 + DC^2} = AC$; $\sqrt{\overline{2.5 \times 2.5} + \overline{6 \times 6}} = 6.5 = AC$. Because A D E F is a regular figure; and, the angles A D E and A F E being equal, each being a right angle, and the sides D E and F E being also equal, because they are



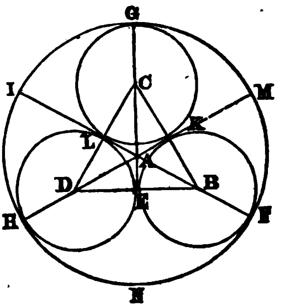
radii of the circle D F H G, the sides A D and A F are also equal. A D is 2.5 inches; A F is also 2.5 inches. If A C be 6.5 inches, and A F 2.5 inches, F C will be 4 inches; $6.5 - 2.5 = 4$ inches. Then, by similarity of triangles (see page 202), C D : D A :: C F : F E; 6in. : 2.5in. :: 4in. : $1\frac{2}{3}$in. = F E. If F E be $1\frac{2}{3}$in., F G will be $3\frac{1}{3}$in. = $\frac{10}{3}$in., because F G is the diameter of the sphere, and F E the radius or semi-diameter. By mensuration of solids (see pages 260 and 262), we find the contents of the cone and sphere in the following manner:

$5 \times 5 \times .785398 \times 2 = 39.2699$in. = contents of the cone;
$\frac{10}{3} \times \frac{10}{3} \times \frac{10}{3} \times .5236 = \underline{19.3925}$in. = contents of the sphere;
19.8774in. = the cubic inches of water that will remain in the cone after the sphere is immersed. Having taken it for "granted," that cones, spheres, and all regular solid bodies, are to each other as the cubes of their homologous sides, we say, As the quantity of water it requires to immerse the sphere in the *given* cone is to the cube of the diameter of the sphere, so is any other quantity of water in the conical glass to the cube of the diameter of a sphere that may be immersed in it. Now, the quantity of water given to immerse the *required* sphere is $\frac{1}{5}$ of the contents of the conical glass $= \frac{39.2699}{5} = 7.85398$ cubic inches. The cube of the diameter of the *given* sphere is $\frac{10}{3} \times \frac{10}{3} \times \frac{10}{3} = \frac{1000}{27}$in. Therefore

$$19.8774\text{in.} : \tfrac{1000}{27}\text{in.} :: 7.85398\text{in.} : 14.634114529 + \text{in.};$$

$$\sqrt[3]{14.634114529} = 2.445 + \text{in. Ans.}$$

34. Let the larger circle of the annexed figure represent the farm of the lady, and the three smaller circles the farms of her daughters. To construct this figure, make the equilateral triangle C B D, each of whose sides is 10 rods or inches. Bisect each of the sides D C, C B, B D,



in the points L, E, K; and draw the lines I F, E G, F I, at pleasure. Upon the points C, D, B, as centres, and with the distance C L, as a radius, describe the circles G L K, L H E, K E F. Upon the point A, as a centre, where the lines I F and H M intersect each other, and with the radius A G, describe the circle G I H N F M, and it will touch the peripheries of the smaller circles without cutting them. The sides of the triangle being 10, the diameter of each of the smaller circles will be 10. Because C D E is a right-angled triangle, C E $= \sqrt{CD^2 - DE^2}$; $\sqrt{10 \times 10 - 5 \times 5}$ $= 8.660254+$; and as C D E and A D E are similar triangles, C E : C D :: D E · D A; that is, 8.660254 : 10 :: 5 : 5.7735027 = A D. If we add H D = 5 to D A, we have the semi-diameter of the larger circle, 5.7735027 + 5 = 10.7735027. By multiplying this last number by 2, we have the diameter of the larger circle, 10.7735027 × 2 = 21.5470054. As the area of a circle may be found by multiplying the square of the diameter by .785398, therefore, by dividing the area by .785398, the quotient will be the square of the diameter. The area of the lady's field is 500 acres = 80000 square rods; 80000 ÷ .785398 = 101859.18 square of the diameter; $\sqrt{101859.18} = 319.154006+$ rods = diameter of the lady's farm. To find the diameter of each of the daughters' farms, we say, As the diameter of the larger circle in the diagram is to the diameter of one of the smaller circles in the diagram, so is the diameter of the lady's farm to the diameter of either of her daughters' farms. 21.5470054rd. : 10rd. : : 319.154006rd. : 148.119889 + rd. = diameter of the daughters' farms; and the distance of their houses from each other, 148.119889 × 148.119889 ×

.785398 = 17231.2406 + square rods in each of the daughters' farms; 17231.24 + rd. ÷ 160 = 107A. 2R. 31.24 + rods = acres, &c. in each of their farms; 107A. 2R. 31.24 × 3 = 323A. 0R. 13.72 amount of the 3 daughters' farms; 500A. — 323A. 0R. 13.72rd. = 176A. 3R. 26.28rd. the lady retained. To find the distance of the lady's dwelling-house from those of her daughters, we subtract the semidiameter of either of the daughters' farms from that of the lady's; thus, 319.154 + rd. ÷ 2 = 159.577 + rd.; 148.119 + rd. ÷ 2 = 74.059 + rd.; 159.577 + rd. — 74.059 + rd. = 85.518 + rd. Ans.

THE END.

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Haverhill, (Mass.) May 22, 1843.

B. Greenleaf, Esq. Dear Sir: We have examined your Arithmetics, the National and Introductory, and take pleasure in expressing to you our high satisfaction in them, as superior to any books in this branch of education with which we are acquainted. We are especially pleased with the accuracy and precision of the definitions, and with the clearness and fullness of illustration by the examples. The two together seem to be just what are needed, and we are inclined to say *all* that are needed on this subject in our Public Schools. In accordance with this view of your books, as members of the General School Committee, we have encouraged their use in the Schools in this town.

(Signed,) EDWARD A. LAWRENCE, } *Superintending*
A. S. TRAIN, } *School Committee.*

Bradford, May 5, 1843.

Benjamin Greenleaf, Esq. Dear Sir: The School Committee of this town, having given the Introduction to your National Arithmetic a pretty thorough examination, very soon after its publication, voted unanimously to introduce it into our schools, and are now, after a year's experiment, happy to say, that our best anticipations have been met, in the manifest advantages which have resulted from its use, and we feel great confidence in recommending it to the attention of an enlightened public, as a work well calculated to aid youth in acquiring a theoretical and practical knowledge of that important part of education. With much respect, I am, dear Sir, yours,

G. B. PERRY,
In behalf of the Committee.

Having used Greenleaf's Arithmetic in the schools with which I have been connected for three years past, I am prepared to give it the preference over any other work of the kind with which I am acquainted.

Very respectfully yours,
A. FARWELL,
Andover, June 6, 1843. *Principal of Abbott Female Academy.*

From Rev. Mr. Shailer, formerly Principal of the Connecticut Literary Institution, Suffield, Ct.

I have somewhat carefully examined the National Arithmetic, by Benjamin Greenleaf, Esq., and though having had considerable acquaintance with other works upon this science, in several years' experience as a teacher, I hesitate not to pronounce this treatise superior to any I have ever seen. It is in my opinion, impossible for a scholar to go through with this work, and understand its rules, without being qualified, so far as Arithmetic is concerned, to engage in any ordinary business, and having a foundation laid for acquiring with rapidity the higher branches of mathematics needed in professional life.

It is sufficient to say, the School Committee of this town have adopted it in all our Public Schools, which is the highest praise that we can give to any school book.

WILLIAM H. SHAILER,
Brookline, June 6, 1843. *Secretary of the School Committee.*

Portland, (Me.) May 22, 1843.

I have thoroughly examined, and used in my School, Greenleaf's National Arithmetic; and gladly do I embrace a favoring opportunity of rendering this too tardy justice to its merits, and of paying a willing tribute to its superior excellence as a *system*, and as a *text-book*.

(Signed,) B. CUSHMAN,
Late Principal of Portland Academy.

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Brookline, (Mass.) June 1, 1843.

Mr. R. S. Davis. Dear Sir : I am glad to have an opportunity to express to you the high opinion, which I entertain of the merits of the National Arithmetic by Mr. Benj. Greenleaf. I have used it nearly two years, part of the time, however, only to obtain examples for practice by my classes, as an Arithmetic by another author was required by the Committee, but lately my classes have used no other, the School Committee, much to my satisfaction, having required this in preference to all others.

The clearness of the Rules, the simplicity of the arrangement, the interesting character of the examples, the addition of the problems in Natural Philosophy and in Geometry, and the beauty of the typography, make this treatise the most complete in all its parts of any which I have ever examined.

Benjamin H. Rhoades,
Principal of Brookline Public High School.

Boston, May 26, 1843.

I have examined Greenleaf's Introductory Arithmetic, and consider it well adapted to the purpose for which it is designed. I have also used it in my School for a short time, and like it much ; it has but recently been introduced, but thus far it has verified all that it promised. The *larger* work I have also introduced to my School, and find the operation to be good. I can cheerfully recommend it as an excellent work for higher classes.

Charles E. Abbott,
Principal of Temple School.

Medford, (Mass.) May 15, 1843.

I have used Mr. Greenleaf's Arithmetic in my School for the last year, and consider it preferable to any other work treating upon the same science, with which I am acquainted.

Stacy Baxter,
Teacher of East Grammar School, Medford.

I fully subscribe to Mr. Baxter's opinion.

Thomas S. King,
Teacher of West Grammar School.

Portland, May 23, 1843.

I have long considered Greenleaf's Arithmetic to be a superior work, and, for the last two years, I have recommended it to my pupils in preference to every other.

The Introduction to the same book, I have just examined with the most favorable impression of its merits.

D. Greene Haskins,
Preceptor of Portland Academy.

From W. R. Ellis, Esq., Principal of Sandwich Academy.

Mr. R. S. Davis. Dear Sir : Before introducing Greenleaf's Introduction to the National Arithmetic into my school, I gave it a careful and thorough examination, and was at that time much pleased with it. And since its introduction, I have become convinced, from comparing the improvement of my pupils in this branch, with that of previous terms, that, as an introduction to the study of arithmetic, teachers cannot place in the hands of their pupils a better book.

Very respectfully, yours,
(Signed,) W. R. Ellis.

Sandwich, (Mass.) March 8, 1843.

Portland, (Me.) July 14, 1843.

Greenleaf's Arithmetic has been so long known, and its superior merits so fully attested by all who have used it in their schools, that were I to say that for clearness and conciseness, and for a general adaptedness to the wants of pupils of both sexes, it stands *unrivalled*, I should only reiterate the opinions expressed by every teacher I have met with, who has had an opportunity of making trial of it. I shall introduce it to my school another term.

EBEN S. STEARNS,
Principal of Free Street Seminary.

Extract from a Letter from A. Mackie, Esq., of Grove School, New Bedford.

"Teachers will find in this book precisely that which they would desire placed in the hands of a scholar, who attends school but for three or four months in the year; they will find that it goes into the subject as far as is necessary to prepare the young for the common avocations of life. They will find it no less practical than the author's larger work, — while the several questions and rules succeed each other in such a manner, as will not perplex and embarrass the student, but, on the contrary, in that inductive and progressive manner, which cannot fail to encourage and stimulate him onward; and, above all, they will find it is calculated to unfold to the scholar the powers of his own mind, which he only needs to know and feel, in order to bring into action. The work is printed with a beautiful type, on good paper, and is indeed got up in a style which reflects great credit on the publisher." (Signed,)

"*New Bedford, Dec.* 6, 1842." "ADAM MACKIE."

From James K. Bullough, Esq., Teacher, Savannah, Ga.

I have with care and much satisfaction, examined Greenleaf's Arithmetic, and deem it a work of peculiar merit. It is in every respect well calculated to facilitate the improvement of pupils in that highly useful and important science, the science of numbers. It is a book much needed in our elementary schools. I shall most cordially recommend it to others of my profession, and shall also use my utmost endeavours to introduce it into my own School.

(Signed,) JAMES K. BULLOUGH.

Savannah, Feb. 1, 1843.

From O. M. Randall, Esq., Teacher, Lynn, Mass.

I have examined the Introduction to the National Arithmetic, and think it admirably adapted to the use of common schools. I have long felt the need of something different from what we have had in our town schools, to give the scholar a practical knowledge of arithmetic, without retarding his proficiency by a multiplicity of questions so arranged as to confuse the mind, rather than unfold to it the principles of the science. I think the Introduction meets the exigency of the case, and cannot fail to secure that patronage which it so richly merits. (Signed,) O. M. RANDALL.

Lynn, (Mass.) Jan. 9, 1843.

Plymouth, (Mass.) March 19, 1843.

Dear Sir: I received, some weeks since, a copy of the introductory, and second part of Greenleaf's Arithmetic.

With the merits of the latter, I long since became acquainted, and have no hesitation in saying, that it is the best work of the kind I have ever seen.

I think, from what little attention I have given to the introductory, that it compares very well with the second part, and shall take great pleasure in seeing them introduced into all our schools. Very respectfully,

PHILIP C. KNAPP,
Principal of the High School, Plymouth.

R. S. Davis, Esq., Boston.

Benjamin Greenleaf, Esq. Dear Sir : I regard your National Arithmetic as one of the best I have ever seen. Perhaps the best proof of the estimation in which I hold its merits, is the fact, that I use it in the school under my care.

I am, Sir, very respectfully, yours,
Roger S. Howard,
Principal of the Latin High School.
Newburyport, May 5, 1843.

I have used Mr. Greenleaf's National Arithmetic in my School for nearly two years ; and, having thus tested its good qualities, I can cheerfully recommend it, as a system of arithmetic well adapted for giving an individual a thorough knowledge of the science.

A. H. Merriam,
Preceptor of Westminster Academy.
Westminster, (Mass.) June 6, 1843.

I have made use of Mr. Greenleaf's National Arithmetic in my school, and am of the opinion, that it possesses superior excellences as an Arithmetic, and well adapted to our common and higher Schools.

F. G. Pratt,
Bridgewater, (Mass.) June 14, 1843. *Preceptor of Bridgewater Academy.*

The undersigned, having examined the National Arithmetic on the Inductive System, by Benjamin Greenleaf, Esq., do not hesitate to pronounce it a work of high merit. The various subjects treated of in it are arranged in a manner at once philosophical and practical ; and, in the opinion of the undersigned, it contains a greater amount of useful and valuable matter, some of which must otherwise be sought for in rare books, than any other similar work with which they are acquainted. And they cheerfully recommend it to teachers and learners, as a work of high and undoubted worth.

Thomas C. Baker,
John P. Pendleton,
John P. Adam,
A. T. C. Dodge,
Superintending School Committee.
Prospect, (Me.) March 1, 1843.

Extract from a Letter from Hiram Orcutt, Esq., Teacher.
Hebron, N. H., Feb. 27, 1843.

"Your Arithmetic I have had opportunity thoroughly to examine, having introduced it into my School, and conducted *two large classes* of teachers entirely through it. And I can freely say, Sir, that in my opinion, no book of the kind now extant, is so well calculated to lead the student to a thorough practical knowledge of figures as this."

New Bedford, Mass., Dec. 26, 1842.

Benjamin Greenleaf, Esq. Dear Sir : We have examined your Introductory Arithmetic, and are much pleased with the plan and execution. The examples are practical ; the rules clear and concise ; the principles of the science are unfolded, and its practical uses explained with great perspicuity and simplicity. We deem it eminently calculated to answer the object for which it is designed.

Benjamin Evans, *Principal of the Charles-St. School.*
Ebenezer Hervey, *do. Sixth-St. do.*
A. L. Gleason, *do. Bush-St. do.*
William F. Dow, *do. William-St. do.*
Albert Conant, *do. Maxfield-St. do.*
Fred. F. Dewey, *do. Hill do.*
Adam Mackie, *do. Grove do.*

A*

I have carefully examined the National Arithmetic you were so good as to send me, and am happy in being able to say, that, in my opinion, it is preferable to any other that I am acquainted with. I recollect that so highly was it esteemed, both by the Committee and myself, that it was immediately after examination introduced into my School as the text-book in that science.

Yours, very respectfully,

D. Worcester,

Principal of the High School for boys, Bangor.

Bradford, (Mass.) January 1, 1842.

Gentlemen: The National Arithmetic, by Benjamin Greenleaf, Esq., after having been pretty thoroughly examined by those entrusted with the superintendence of the Public Schools, was introduced into them very soon after its first publication, and has been constantly used from that time; and, after so long and extended experiment, I have no hesitancy in saying, the best expectations of the committee have been fully met. A new and increased interest was thereby given to that part of education, which has been constantly increasing in most or all our Schools, — a spirit of perseverance excited, which has carried a considerable number of our youth through the whole book, and there are now a still greater number, who are going on with a determination to perform every question. Having formerly recommended the work, as, in my opinion, possessing great merit, from the lucid and scientific manner in which its parts are arranged, and the rules of operation expressed, I have thought the above statement would afford the best testimony that I could now give of its practical worth, and of the benefits likely to result from its introduction into other places.

Very respectfully,

(Signed,) Gardner B. Perry.

From H. Morrison, Esq., Professor of Mathematics and President of the University of Maryland; Baltimore.

This is one of the most complete books of its kind, both in the extent and arrangement of its matter, that has yet appeared. Combining, as it does, the Analytic and Synthetic methods, and abounding in familiar examples, it is admirably calculated to interest the pupil, and lead him, by easy and progressive steps, through the difficulties of the science, to its complete mastery, and full comprehension. To make the work more perfect than a treatise on arithmetic merely could be, the author has added many geometrical, mechanical, philosophical, and astronomical problems, and a concise system of book-keeping, so that without the aid of any other book, it is calculated to make the perfect business man, in all his various departments. H. Morrison.

I cheerfully join with Prof. Morrison in recommending the National Arithmetic to the favorable notice of teachers. H. Colburn.

Extract from a Letter from Mr. D. H. Armstrong, Teacher, St. Louis.

I am gratified to be able to give it my unqualified approbation, as a work justly meriting high commendation for its simplicity of arrangement, its clearness of illustration, its fullness of important matter to the mercantile student, recently deemed wholly useless, or at best irrelevant to the subject of arithmetic, and for the variety and copiousness of its examples.

D. H. Armstrong.

From D. F. Merrill, Esq., Teacher of a Classical School in Mobile.

I have carefully examined Greenleaf's National Arithmetic, and am much pleased with it. The arrangement is good, the rules and explanations are clear and concise, and the numerous questions are well selected. It is, in my opinion, superior to any arithmetic with which I am acquainted, and I shall use it to the exclusion of all others. D. F. Merrill.

Norfolk, (Va.) January 12, 1842.

A careful examination of Greenleaf's National Arithmetic, will show, that its author has compiled it, as all books ought to be, from the results of actual experiment, and observation in the school-room. It is entirely a practical work, and appears to me the most complete system of mercantile arithmetic, with which I am acquainted. The brief system of book-keeping attached to it, will be a valuable aid to more complete instruction in Common Schools, to which the work is, in other respects, so peculiarly adapted. Being so much pleased with it, I will introduce it into my Academy.

JOHN P. SCOTT,
Principal of Norfolk Academy.

I fully concur in the opinion above expressed.

ROBERT WELLS,
Professor of Mathematics in Norfolk Academy.

Dear Sir: I thank you kindly for the copy of Greenleaf's Arithmetic which you presented to me a few days since. I have examined it with great care, and am convinced that it is decidedly the best work within my knowledge. As the highest proof I can give of my approbation of the work, I have adopted it, and have given orders for a supply sufficient for the use of this school.

Yours, &c. J. WORTHINGTON SMITH.
Kalorama Seminary, Staunton, (Va.) April 28, 1842.

Having been adopted recently in many select schools in New York city and vicinity, the following testimonials from teachers, will show the estimation formed of the work, by those who are best qualified to judge of its value.

After a careful and critical examination of Greenleaf's Arithmetic, being highly impressed with its merits, I determined to introduce it into my School, and, accordingly, since last fall, have used it, with entire satisfaction, as a text-book, considering it the best treatise on figures with which I am acquainted.

(Signed,) JAMES LAWSON,
101, *Grand Street.*

I have examined Greenleaf's National Arithmetic with much minuteness, and without hesitation, give it the preference to any work of the kind with which I am acquainted. I should be pleased to see it generally used in our schools. I have already placed it in the hands of my pupils.

(Signed,) E. H. JENNY, A. M.,
Principal of Seventh Ward Grammar School.

Communications fully agreeing with the foregoing, have been received from the following Teachers, who now use this Arithmetic in their respective schools.

G. A. Rogers, } St. Luke's School.
J. Yeamans, Jr. }
Henry Swords, 88 Avenue Sixth.
M. Beardsley, Corner of Houston and Thompson Streets.
John Mulligan, Principal of St. Mathew's Academy.
Dr. Edward Kupperbery, Teacher of Mathematics in do.
John Walsh, Professor of Languages, &c.
Nath. H. Arey, Grand Street.
A. Bassett, Principal of the Classical and Mathematical Ins., 472½ Broad.
Charles Wm. Nichols, Allen Street.
Robinson and Finch, Grand Street.
S. R. Martin, D. Lyme, R. Lockwood.

Petersburg, (Va.) Jan. 25, 1842.

In the cursory examination that I have been enabled to give to the National Arithmetic, I have discovered excellences possessed by no other arithmetic with which I am acquainted.

The logic of the system is as commendable for what it omits as for what it retains, being such as to furnish healthy exercise to the mind, without embarrassing it with subtleties, or enervating it by secure indolence.

It is the very book that I have long felt the want of, and I shall not fail to recommend its introduction into the Seminary over which I have the honor to preside. J. D. KEILY, A. M.,
Principal of the Anderson Seminary.

The subscriber has examined with attention the National Arithmetic, compiled by Benjamin Greenleaf, Esq., and has introduced it into his School, being confident in the opinion, that the judicious arrangement of the rules, and the manner of its execution, justly entitle it to general patronage.
(Signed,) JOSHUA HEALY,
Teacher, Brooklyn.

I fully concur with Mr. J. Healy respecting Greenleaf's Arithmetic.
WM. M. MARTIN,
66 *Cranbury Street, do.*

Also adopted and recommended by Alfred Greenleaf, Esq., Principal of the Young Ladies' School, Brooklyn, N. Y.

I most cheerfully concur in the preceding opinions expressed of this Arithmetic, and cordially recommend it to teachers and others, as a work of great utility. (Signed,) ALBERT T. SMITH,
Principal of Jersey City Public School, No. 1.

From Rev. D. Leach, Principal of Roxbury High School.

I have examined with care and attention Greenleaf's National Arithmetic, and am happy in having an opportunity of expressing my opinion of its excellence. It is truly a very complete and practical work, — one of the best, if not the very best I have ever seen. I have also examined the Introduction, but with less attention, and am highly pleased with its judicious arrangement, concise rules, and practical character. The National Arithmetic has lately been adopted in my School. Very respectfully, yours,
Roxbury (Mass.) High School, June 12, 1843. DANIEL LEACH.

Having used Mr. Greenleaf's National Arithmetic for several years in my School, I am decidedly of the opinion it is the *best* work of the kind now extant. JOHN T. TASKER,
Principal of the Male High School, Portsmouth, N. H.

After a careful and candid examination of Mr. Greenleaf's National Arithmetic, I think I can truly say, that, in regard to variety and amount of useful matter, skill in arrangement, and clearness of illustration, it excels all other works of the kind with which I am conversant. It happily unites the inductive with the synthetic methods of instruction, and embraces several rules and tables commonly omitted in similar treatises. It is enriched with an excellent article upon the subject of exchange, and it also contains a new and concise system of book-keeping, which, together with a short treatise on geometry, greatly enhances its value. Indeed, I am acquainted with no system of arithmetic so complete and practical as this, and I shall therefore introduce it immediately into my School. ELIAS NASON,
Principal of the Young Ladies' High School, Newburyport.

Having made constant use of the National Arithmetic for five years past, I gladly embrace an opportunity of expressing the high opinion which I entertain of its merits. In my view, it is decidedly preferable to any other work of the kind with which I am acquainted.

(Signed,) **Oliver A. Woodbury,**
Salem, (Mass.) July 7, 1843. *Teacher.*

From Mr. R. S. Harlan, Teacher, Baltimore.

After an examination of Mr. Greenleaf's Arithmetic, which you politely sent me, I have no hesitation in pronouncing it superior to most works of the kind; and by its concise and plain rules, well adapted to the purposes for which it was designed. This Arithmetic has very many things to recommend it to the careful consideration of teachers, and trustees of schools generally.

Yours, respectfully, **R. S. Harlan.**

Having examined Greenleaf's National Arithmetic, I do not hesitate to say, that for its simplicity and systematic arrangement, I think it excels every other publication on the subject with which I am acquainted. I intend to introduce it into my School soon. The key is also a valuable work.

P. M. Neal,
Teacher of Select School, Portland, Me.

From a careful examination of Greenleaf's National Arithmetic, my impressions are, that it possesses more merit as a school book for general and practical use, than any other before the public. I shall use it next term in my School.

Ahira Jones,
Principal of the High School, Saco, Me., and Superintending School Committee of the Town.

From A. B. Converse, Esq., Bangor.

A copy of Greenleaf's National Arithmetic was handed me a few days since, which I have perused with some care. It is already introduced into some of our schools, and I do not hesitate to say, I think it the best we have in use.

Respectfully, yours, **A. B. Converse.**

My Dear Sir: I have examined the National Arithmetic, by Mr. Greenleaf, with care, and have used it two or three years in my School. I consider it decidedly the best system of arithmetic with which I am acquainted.

It is a *complete system*, and admirably adapted to facilitate the progress even of the youngest pupil, and to make thorough scholars in the science it teaches.

The Key is a valuable aid to the teacher, and, as the author intended, should be in his hands alone. **Alfred W. Pike,**
Principal of the High School, Hallowell, Me.

From Isaac Candler, Esq., Teacher, Baltimore.

Having examined Greenleaf's National Arithmetic, I am able to inform you, that it combines what is termed Mental Arithmetic, with the more complete branches of the science, thus leading youths to be ready, as well as exact, in the various operations. But lest they should perform them merely mechanically, as the generality of persons do, the principles of the rules are given, by which means they will be better grounded, than by doing double the work, without understanding the reasons for the mode of procedure. The compendiums of book-keeping and geometry appended to the treatise, render the whole particularly eligible for schools.

I shall take pleasure in recommending the work, it being as superior to some of the Arithmetics now in use, as Adams's Latin Grammar to the Eton, or as Webster's Dictionary to Bailey's. **Isaac Candler.**

GREENLEAF'S NATIONAL ARITHMETIC.

From Mr. J. P. Engles, A. M., Principal of the Classical Institute, Philadelphia.

I have examined, with considerable interest, Greenleaf's National Arithmetic, and have no hesitation in recommending it as an admirable system of Arithmetic, which contains all that is essential to a knowledge of the science, and nothing that is useless. The arrangement, too, is such as to make the contents easily available to the teacher and the pupil. Should it succeed in displacing the host of so called "Assistants," with which our schools are flooded, I conceive it would be equally to the comfort of teachers, and the profit of students. I shall cheerfully introduce it into my Academy. J. P. Engles.

Philadelphia, Nov. 14, 1838.

I cheerfully concur in sentiment with Mr. Engles, respecting Mr. Greenleaf's Arithmetic; it is the best work of the kind I have ever seen. With a great deal of pleasure, I shall introduce the same into my Seminary.

W. Alexander, *Classical Teacher, Philadelphia.*

I have examined Greenleaf's National Arithmetic with a great deal of satisfaction, and have no hesitation in saying, that it is the most complete system of Mercantile Arithmetic with which I am acquainted; and will cheerfully recommend it as occasion may require.

E. Griffiths, *Teacher of Mathematics, Philadelphia.*

Philadelphia, Nov. 12, 1838.

The undersigned entirely concur in the opinions expressed by Messrs. Engles, Alexander, and Griffiths, respecting Mr. Greenleaf's Arithmetic.

John W. Faires,
B. P. Hunt,
James P. Espy, } *Teachers in Philadelphia.*

I have examined Mr. Greenleaf's National Arithmetic with some care, and am much pleased with its arrangement; his examples, under each rule, are numerous and appropriate: I am so well satisfied, that I intend to introduce it into my Seminary. Thomas McAdam.

Philadelphia, Nov. 14, 1838.

We fully concur with the gentlemen, who have already given recommendations of the National Arithmetic, considering the work well calculated to give youth a correct knowledge of the principles of Arithmetic.

Wm. Vogdes,
E. O. Kendall, } *Philadelphia Centre High School.*

Copy of a letter from G. W. Harby, Esq., Principal of Harby's Academy, New Orleans, addressed to the Publishers.

Gentlemen: Viewing the publication of School Books of the first importance, it was with much pleasure that I received Greenleaf's National Arithmetic. For fifteen years, and upwards, I have devoted my life to the instruction of youth, during which time many Arithmetics have fallen under my inspection. I take a strong interest in every work that pertains to mathematical learning, and unhesitatingly pronounce Greenleaf's Arithmetic an important treasure to Academies; it is fraught with a great deal of care, and in an easy, plain, and uniform style. His Geometrical, Mechanical, and Astronomical Problems are concise and clear: they lead the youthful mind to the exercise of a little patience,—not so arduous as to fatigue, but sufficiently laborious to call the mental faculties into exercise, and to create a taste for mathematical knowledge, and for scientific discovery and invention,—which has lately so conspicuously crowned some of our countrymen with brilliant success. I shall make it the standard book in my Institution, and recommend it to others of my profession.

I remain, gentlemen, your obedient servant,

New Orleans, August 22, 1839. George W. Harby.

Portsmouth, Aug. 5, 1838.

Benjamin Greenleaf, Esq. Dear Sir: Having examined, and, to some extent, introduced into our Schools the National Arithmetic, of which you are the author, we deem it a duty we owe to the public, no less than to yourself, to express our decided approbation of its merits. The method, arrangement, and quantum of matter it contains, the clear and lucid manner in which its rules are demonstrated, together with its adaptation to the wants of the community, entitle it, in our humble belief, to the patronage of every lover of scientific investigation

Signed,

HAZEN PICKERING, JOHN T. TASKER,
A. M. HOYT, JOHN J. LANE,
JAMES HOYT, EDWARD J. LAIGHTON.
C. E. POTTER,
School Teachers of Portsmouth, N. H.

From Rev. Dr. Hopkins, President of William's College.

My opinion of Greenleaf's Arithmetic is, that it is adapted to give a more thorough knowledge of that science, than any other that I have seen.

Respectfully, yours, M. HOPKINS.
Williamstown, Dec. 30, 1837.

Poughkeepsie Institute, Jan. 9, 1839.

We have carefully examined the National Arithmetic, and do not hesitate in pronouncing it the best work of the kind which has come under our notice. The deduction of the rule from the operations is, in our opinion, the proper method; and the copious examples, under the various rules, are well selected and arranged. We hope it may meet with its merited success. We shall endeavour to extend and establish its use.

Yours, respectfully,
J. L. DUSINBERY, } *Principals.*
A. H. TOBEY, }

I have examined Greenleaf's Arithmetic, and consider it, in many respects, preferable to any work of the kind with which I am acquainted. I am particularly pleased with his illustration of the Square and Cube Roots, and the Rule of Proportion, and with the introduction of practical instruction on the subject of Banking, Custom-House Duties, Assessment of Taxes, &c. I think its introduction into Schools and Academies will prove of general interest to all who wish to acquire a knowledge of Arithmetic.

A. B. BULLOCK.
Hudson, Dec. 7, 1838.

I fully concur in the above, and shall use my influence to introduce it into my school.

C. GREENE.

From the Principal of the Dutchess County Academy.

After a careful and comparative examination of Greenleaf's Arithmetic, I unhesitatingly say, I think it superior to any other Arithmetic within my knowledge. I shall with pleasure use my influence to give it a circulation in the Schools of this vicinity.

WM. JENNOY.
Poughkeepsie, Jan. 9, 1839.

I fully accord with Mr. Jennoy in his opinion of Mr. Greenleaf's Arithmetic, and shall esteem it a privilege to recommend its use whenever an opportunity presents.

O. M. SMITH,
Newburgh, January, 1839. *Principal of Newburgh High School.*

GREENLEAF'S NATIONAL ARITHMETIC.

I have examined Mr. Greenleaf's Arithmetic, and think it the best that has come under my notice. We shall introduce it into our School.
Newburgh, (N. Y.) Jan., 1839. SAMUEL PHINNEY,
Principal of the Orange County Institution.

I have examined Mr. Greenleaf's Arithmetic, and find it an excellent work; the Rules are very plain and distinctly in order, and the Examples are most excellent, and ought to be patronized in all Schools. I shall use all my endeavours to have it used in my School. JOHN B. CLUTE.
Schenectady, Nov. 29, 1838.

I fully subscribe to the above opinion. R. M. BROWN
Schenectady Lyceum, Dec. 3, 1838.

From Rev. Wm. Cogswell, D. D., Cor. Sec. of the American Education Society.
Boston, March 16, 1838.

B. Greenleaf, Esq. Dear Sir: I take this early opportunity to acknowledge the receipt of your National Arithmetic, and to thank you for it. I have cast my eye over it as my time would permit, and am very happy to say, I have been much gratified in its perusal. The inductive plan, combining the Analytic and Synthetic methods of instruction, which you have adopted, is undoubtedly the best. The work contains a very great amount and variety of matter for its size, and is judiciously arranged. Its Rules, Explanations, and Examples are perspicuous, copious, and apposite, and some of them are ingenious and original. It is a treatise of much merit, exhibiting indefatigable labor, and great practical skill in the science and in the art of teaching this branch of an education. The type and execution of the book are good, and do honor to the publishers. As it becomes known to the public, its excellence, I trust, will be fully appreciated, and your unwearied application of mind to this department of instruction, for thirty years past, in the results here given, be duly rewarded. Yours, with much respect.
WILLIAM COGSWELL.

Troy, (N. Y.) Nov. 28, 1838.

Dear Sir: I have given the National Arithmetic a second examination, am much pleased with its system and arrangement, am inclined to believe, that, if its merits can be generally known, it will supersede the use of about all others now in use.

I have now in my School eight different kinds of Arithmetics, have frequently regretted, that it should take so many to make one, and that *one* not complete.

The National, I consider, possesses all the good qualities of these eight, with some additions that are entitled to patronage, and all in *one book.*

I do sincerely hope, your labors in this valuable work will be generously rewarded, for the benefit of its author and the public good.
Yours, respectfully, L. E. GIBBS, *Teacher.*

I have examined, with much pleasure, a text-book, entitled Greenleaf's National Arithmetic. In many respects, I think it superior to the Arithmetics in common use. The Explanations are very lucid, and the Examples of such a character as to lead the pupil to make a practical application of every thing he learns. The articles on Exchange, Roots, the System of Book-Keeping, &c., are calculated to be eminently useful to the business man. As a whole, the National Arithmetic is equal, if not superior, to any work of the kind that I have ever seen. C. H. AHTHONY,
Troy, Dec. 31, 1838. *Principal of Troy Academy.*

GREENLEAF'S NATIONAL ARITHMETIC.

I have examined, with considerable care and entire satisfaction, the System of Arithmetic by B. Greenleaf. I can say, without hesitation, I think it the most complete and well-arranged School System, in this branch of science, extant, and better calculated than any other to prepare our youth for active usefulness in all those pursuits where a knowledge of Arithmetic is requisite. I might speak of the happy combination of the Analytic and Synthetic methods of operation, and the still happier union of clearness with brevity in all the Rules and Definitions ; but all this will be seen and pleasingly felt by those who peruse or study this truly valuable book. I shall do what I may, in my limited sphere of influence, to promote its introduction into the Schools of our State.
Albany, Dec. 1838. S. Steele, *Teacher.*

I have examined Greenleaf's National Arithmetic, and am of opinion, from its practical character and the order of the arrangement, that it is well calculated to induct the inquiring pupil into the useful business operations of the community, for which the study of Arithmetic is designed. I shall not hesitate to recommend it to my own pupils and to the teachers of other Schools.
Edward Small,
Albany, Dec. 1, 1838. *Teacher of the Lancaster School, Albany.*

Mr. Greenleaf. Sir : I have examined your National Arithmetic and am glad to say, it meets my approbation ; and I think I shall introduce it into my School, to the exclusion of all others. A. P. Smith,
Albany, Nov. 28, 1838. *Teacher of the Second Public School, Albany.*

Mr. Greenleaf. Dear Sir : I have examined your System of Arithmetic, and am happy to state, that it meets with my unqualified approbation, and that I shall immediately introduce it into my School. Yours, respectfully,
Albany, Nov. 27, 1838. Thomas McKee.

We fully concur in the above.
Newman & Wallace, *Teachers, Mechanics Academy, Albany.*
D. E. Bassett, *Principal of an Academy, Do.*
Joel Marble, *Principal of District School, State Street, Do.*
J. W. Bulkley, *Principal of an Academy, Do.*

From Dr. Fox, Principal of the Boylston School, Boston.

B. Greenleaf, Esq. Dear Sir : I have just been examining your new Arithmetic, and think it an excellent work. I like the plan of it much. Among its many excellences I perceive the following, viz. — The Tables of Money, Weights, and Measures carried out to the lowest denomination ; the great variety of examples under each Rule, and likewise your method of treating several parts of the science, as Fractions, Proportion, Evolution, and Exchange, — every thing concerning them must appear clear, I think, to the student. The Geometry, Philosophical Problems, Mechanical Powers, and Book-keeping, seem also to be handled in a perspicuous manner. The Rules of Cross Multiplication and Position, I am happy to see have place in the work ; for, after all, they are too useful, the latter especially, to be omitted in our arithmetical treatises On the whole, the work appears to me well calculated to lead youth to a clear and thorough knowledge of the various branches of this Science, and I doubt not it will be sought after, as an improvement on former works of the kind, and obtain an extensive circulation. Yours, respectfully, Charles Fox.

A thorough examination of Mr. Greenleaf's Arithmetic has induced me to introduce it into the Academy with which I am connected. The arrangement is excellent, and much valuable matter is found in the National Arithmetic, not contained in others now in use. Very respectfully, yours,
Barnstable, Dec. 9, 1837. F. A. Choate.

From D. P. Page, Esq., Principal of the English High School, Newburyport.

Benjamin Greenleaf, Esq. Dear Sir: I have with much care examined the National Arithmetic, of which you are the author, and, after having compared it, *article by article*, with the various other publications that have come to my hands, I hesitate not to say, that I think it contains a greater amount of matter, and a better arrangement of subjects, than any other book I have seen. Your rules and explanations are clear and definite, and your examples are well calculated to fix them in the mind. I congratulate the community on this valuable accession to our list of school books; and shall take pleasure in seeing your Arithmetic extensively introduced into all our schools, as also into that under my own care. Yours, with just respect, DAVID P. PAGE.

From the late Principal of the Young Ladies' High School, Boston.

Dear Sir: I have examined with great care Mr. Greenleaf's National Arithmetic, and have used it as a text-book for my pupils. In my view, the plan and execution of the work are quite perfect, the rules being deduced analytically from examples, and followed by copious questions for practice. The pupil can hardly fail to *understand* as he advances; nor can he go through the book, without being a master of the science of Arithmetic. This is not an old book with a new name, but the work of one who thoroughly understands the subject, and who has learned, from a long and successful experience in teaching, how to prepare one of the very best school books which has ever been issued from the American press. Very respectfully, E. BAILEY.

Having for two or three years past, made constant use of Greenleaf's National Arithmetic in my School, I am prepared to say, that it is far superior to any work I have ever used.

It appears to me to be a *complete system*, and well calculated, not only to interest the pupil, but also to give him a thorough knowledge of the science. I think it richly deserves the high commendation and liberal patronage which it generally receives.

ALFRED M. HOYT,
Inst. Male School, Portsmouth, N. H.

I have had the National Arithmetic, by Benjamin Greenleaf, in use in my Seminary for several months past, and take pleasure in recommending it as an excellent work.

I have no hesitation in saying, that I not only think it the best single volume on the science of arithmetic extant, but that I consider its value to be equal, if not superior, to that of any *series of arithmetics* now before the American public.

D. RING,
Principal of the East Baltimore Female Institute.

From J. Peckham, Esq., Teacher, Westminster, N. H.

B. Greenleaf, Esq. Sir: I take great pleasure in recommending your National Arithmetic. A number of classes went through with the book in the course of my teaching, and I feel satisfied that they obtained a more thorough and practical knowledge of the science, than they would have done by any other text-book with which I am acquainted. While the work is sufficiently compendious and cheap for general use, it at the same time, fully illustrates every principle in common business. I think the appendix on book-keeping a very valuable addition to the Arithmetic. Your obedient servant,

JOSEPH PECKHAM.

☞ On reference to the "*Abstract of the Massachusetts School Returns*," for 1840, it will be perceived, that Greenleaf's National Arithmetic is used in many of the best Schools and Academies in the State. And wherever teachers have given this system a fair trial, the result has been highly satisfactory.